Optimalité de la dette publique dans une économie à marchés incomplets

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Note de synthèse

Introduction

Les trente dernières années ont connu une augmentation sans précédents du niveau de la dette publique que ce soit en termes absolu ou relatif. D'après les chiffres fournis par les rapports de l'OCDE (OECD Economic Outlook 1985 et 2008), le ratio de dette publique nette rapportée au PNB est passé de $20,7\%$ en 1975 à 33% en 1985 et a continué à croître pour atteindre $43,6\%$ en 2005. Ce fait empirique contraste nettement avec la circonspection que l'on trouve habituellement dans l'histoire ou la théorie économique au sujet du niveau de dette optimal d'un pays. La plupart des auteurs classiques par exemple avançaient que le budget gouvernemental devait être géré de la même manière que le budget d'une famille et que l'existence de dette publique entraînait des dépenses superflues. La parenthèse keynésienne a constitué une des seules périodes où l'accumulation d'une dette publique positive a été considérée comme souhaitable. Rapidement, les mécanismes keynésiens ont été critiqués et l'apparition de la notion d'équivalence ricardienne a évincé la notion de dette optimale. Le cadre de référence qui a prévalu depuis est le modèle de croissance déterministe standard avec un agent représentatif. Dans ce type de modèle, en présence de taxes forfaitaires, il n'y a pas de rôle pour la dette publique. Si les taxes créent des distorsions, la dette publique peut être un moyen de les lisser dans le temps. Cependant, en général, le niveau de dette dépend du niveau initial ou est indéterminé (Barro(1979), Chamley (1985) ou Chamley (1986)). Toutefois, dans la dernière décennie, de nombreux efforts ont été faits pour accélérer la *micro fondation* des modèles macroéconomiques. La littérature a tenté de mieux comprendre les effets de l'intermédiation financière, de l'hypothèse de complétude des marchés ou de l'existence de risque idiosyncrasique sur les politiques fiscales et la gestion de la dette publique. Cette thèse de doctorat peut être rattachée à ce mouvement. Etant donné que le point central de ce travail concerne la dette publique et son niveau optimal dans un cadre où les comportements individuels comme le risque idiosyncrasique, l'interaction avec l'intermédiation financière à travers les contraintes de crédit et l'impact sur les mécanismes d'assurance à travers l'incomplétude des marchés, les résultats et intuitions développés ici doivent être contrastés avec les résultats traditionnels de la littérature. Le cadre de cette thèse se rattache directement à une branche de la littérature créée par Bewley (1980 et 1983) puis Huggett (1993) et Aiyagari (1994).

Objectifs de la thèse

Une des premières motivations ayant conduit à l'écriture de cette thèse était d'apporter une analyse de la dette publique optimale en se détachant du modèle standard à agent représentatif et marchés complets. L'apport des modèles avec agent représentatif en marché complet est très important et fournit des intuitions essentielles pour nombres de questions économiques. Toutefois, il existe également des inconvénients[1]. Un point important en ce qui concerne les politiques économiques est qu'un modèle à agent représentatif ne permet pas une analyse fine des éléments redistributifs. L'hypothèse de marchés complets pose également des problèmes. Sous cette hypothèse, il existe un contrat et un marché pour assurer tout risque futur que l'agent peut rencontrer. Cette hypothèse n'est pas simplement irréaliste. Elle évince également un comportement important et soutenu empiriquement : l'épargne de précaution. Si les agents ne disposent pas d'un système de marchés complets, ils utilisent les moyens dont ils disposent afin de transférer des actifs vers le futur à des fins d'assurance. Le plus souvent, les agents utilisent l'épargne privée à cet effet. De ce fait, sous l'hypothèse de marchés incomplets on observe une hausse de l'accumulation de capital privé. Toutefois, même si les marchés sont incomplets, si les agents peuvent emprunter indéfiniment, ils peuvent également assurer par ce biais le risque futur. L'introduction d'une contrainte de crédit permet de résoudre cette difficulté et correspond à la réalité du secteur de l'intermédiation.

Les recherches empiriques montrent que 10% à 20% des ménages américains sont contraints sur la liquidité (Hall et Mishkin (1982), Hayashi (1985), Mariger (1986), Hubbard et Judd (1986), Jappelli (1990) ou Cox et Jappelli (1993)). Une contrainte

[1] Pour une critique des modèles à agents représentatifs voir Kirman (1992).

de crédit est anticipée par les agents et représente une situation dans laquelle ceux-ci ne souhaitent pas se trouver du fait des effets néfastes sur la consommation et l'épargne. Ceci donne un fort motif d'épargne de précaution. L'incomplétude des marchés est également un ingrédient important pour la compréhension des comportements de consommation et d'épargne individuels. Carroll (1997) souligne que 43% des individus ayant participé à l'enquête Survey of Consumer Finances de 1983 ont répondu que leur motif principal d'épargne était de se prémunir contre les éventualités futures. En contrepartie, seuls 15% des interrogés ont répondu que la retraite était le premier motif d'épargne. Cette étude et d'autres montrent qu'il n'existe pas un système de marchés complets accessible aux agents. Dans la plupart des cas, non seulement les marchés sont incomplets mais l'utilisation des contrats disponibles et l'épargne privée ne permettra que d'assurer assez partiellement le risque futur. Dans un tel cadre l'Etat peut faire office d'assureur. Barsky, Mankiw and Zeldes (1986) le démontrent dans un modèle ricardien à la Barro modifié pour introduire de l'incertitude sur le revenu futur et des taxes proportionnelles. Dans un tel cadre, une baisse courante des taxes constitue une richesse certaine alors que l'augmentation future de l'impôt est soumise à l'évolution future de son revenu. Le fait que le taux de taxe soit proportionnel permet une contribution future totale moins élevée d'un ménage modeste comparée à un ménage aisé. Ainsi, cette politique gouvernementale réduit la variance du revenu futur et permet de fournir une assurance que les marchés sont incapables de proposer. Si les marchés étaient complets, de tels ajustements seraient superflus.

Le comportement d'épargne de précaution quant à lui est bien documenté et peu critiqué dans la littérature. Ce concept, qui apparaît déjà dans le motif de précaution de Keynes (1936), a été utilisé pour répondre à une incohérence du modèle de cycle de vie standard. La théorie standard du cycle de vie prédit que la consommation courante est reliée uniquement aux variations non anticipées du revenu futur. Ceci est infirmé par la littérature empirique (Flavin (1981) ou Hall et Mishkin (1982)). L'épargne de précaution permet de réconcilier la théorie et les observations empiriques et constitue un comportement central des agents pour s'assurer contre le risque futur (voir Zeldes (1989), Kimball (1990) ou Carroll (1997)). Cependant, l'épargne de précaution a également un coût qui est le renoncement à la consommation présente. Ce coût peut être réduit si le taux d'intérêt qui rémunère l'épargne peut être élevé. De ce fait, les agents pourraient épargner moins pour une même couverture du risque et améliorer leur bien-être. Les variations du niveau de la dette publique dans la famille de modèles considérée dans cette thèse permet justement de modifier le taux d'intérêt, puis de modifier le comportement d'épargne de précaution ainsi que son coût et de jouer sur le bien-être des agents.

La seconde motivation ayant entraîné l'écriture de cette thèse était de fournir une analyse quantitative du niveau de dette optimal et de fournir les mécanismes clés, même si l'exhaustivité semblait difficile, ayant un impact sur lui. Le travail majeur d'Aiyagari et McGrattan (1998) a constitué le point de départ de cette volonté. Ce papier quantifie pour la première fois le niveau de dette optimal dans une économie à agents hétérogènes subissant une contrainte de crédit et où les marchés sont incomplets. Etalonné sur les Etat-Unis, ce modèle montre qu'un ratio de dette sur PIB de 66% est optimal. Ce papier décrit également un ensemble de mécanismes pour comprendre l'impact de la dette publique sur les comportements des agents. Cette thèse s'inscrit dans la continuité d'Aiyagari et McGrattan (1998) et fournit une réponse aux limites rencontrées dans ce travail de référence.

Problématique et plan de thèse

Cette thèse a pour ambition d'évaluer le niveau optimal de la dette publique dans une économie à agents hétérogènes subissant une contrainte de crédit et où les marchés sont incomplets. Elle étend la littérature sur ce sujet à travers trois contributions originales. Dans le premier chapitre, nous cherchons à analyser l'impact des fluctuations macroéconomiques sur le niveau de dette optimal dans une économie à marchés incomplets. Woodford (1990) puis Aiyagari et McGrattan (1998) montrent que l'incomplétude des marchés et l'existence d'une contrainte d'endettement à l'origine d'un comportement d'épargne de précaution réaffirme le rôle de la dette publique. Parce qu'elle permet aux ménages de réduire le coût de l'épargne de précaution, et ainsi d'assurer un meilleur lissage de la consommation, l'introduction de la dette est bénéfique dans ce type d'économies. Cependant, ces analyses ignorent une source non négligeable d'accentuation du risque idiosyncrasique de revenu non assurable qu'est la présence de fluctuations macroéconomiques. L'introduction du risque non assurable dans la littérature sur le coût des fluctuations a conduit à reconsidérer les conclusions de Lucas (1987). Krusell et Smith (2002) ou Storesletten, Telmer et Yaron (2001) trouvent, dans des modèles où les fluctuations macroéconomiques sont corrélées au risque idiosyncrasique, que le coût des fluctuations pourrait être plus élevé que ce que rapporte Lucas (1987). Ce coût des fluctuations pourrait, comme le suggère Imrohoroglu (1989), être réduit par des mesures de politique économique. Le chapitre se propose d'explorer cette suggestion du point de vue de la dette publique.

Le second chapitre évalue l'impact de la prise en compte du risque entrepreneurial sur le niveau de dette optimal dans une économie à marchés incomplets. L'article d'Aiyagari et McGrattan (1998) met en évidence qu'il peut être optimal d'avoir un ratio dette sur PIB positif pour l'économie. Cependant, ce résultat s'appuie sur le comportement d'épargne de précaution des agents dans une économie où seul l'emploi salarié est autorisé. Les résultats empiriques montrent qu'une fraction importante de l'épargne nationale est réalisée par les ménages entrepreneuriaux. Ces agents constituent une petite fraction de l'économie mais perçoivent aux Etats-Unis 22% du revenu national et possèdent 40% de la richesse totale. Par ailleurs, ces agents ont un portefeuille très peu diversifié avec une forte composante investie dans leur entreprise. Les travaux de Gentry et Hubbard (2004) et Covas (2006) montrent que face au risque subi par le entrepreneurs, ceux-ci utilisent les autres actifs dont ils disposent comme moyen d'assurance et de ce fait constituent une épargne de précaution. Dans ce chapitre nous montrons que la prise en compte d'agents entrepreneuriaux est une étape importante pour comprendre les effets de la dette publique dans une économie à marchés incomplets. La prise en compte de l'emploi salarié seul peut mal estimer le niveau de dette optimal et la réponse des entrepreneurs aux variations de la dette peut être très différente des agents employés. Enfin, il est important dans la discussion sur le niveau de dette optimal d'avoir une bonne reproduction de la distribution des richesses de l'économie. En effet, comme le suggère Ball et Mankiw (1995) ou Floden (2001), les gains et les pertes liés aux variations de la dette publique sont répartis différemment selon la richesse d'un agent. Les modèles avec une composante entrepreneuriale permettent de reproduire correctement la distribution des richesses.

Le troisième et dernier chapitre s'intéresse au choix de la dette optimale dans le temps. Choisir le niveau de dette optimal dans le temps exige que l'on s'intéresse à la question de l'engagement de la puissance publique. Un pan récent de la littérature, mené par les papiers de Krusell (2002), Klein et Rios-Rull (2002) ou encore Klein, Krusell et Rios-Rull (2007), s'intéresse aux politiques publiques en l'absence de possibilité d'engagement puisqu'il n'existe pas de méthode parfaite pour garantir cet engagement. De telles politiques permettent d'appréhender la cohérence temporelle de la puissance publique et se réfèrent à des gouvernements successifs jouant un jeu contre eux-mêmes dans le temps. Afin de comprendre la raison pour laquelle une majorité d'Etats entretiennent un niveau de dette important, Krusell, Martin and Rios-Rull (2006) reprennent le modèle de Lucas et Stokey (1983) et relâchent l'hypothèse d'engagement. Leurs résultats ne permettent pas de souligner en général qu'un niveau de dette élevé est l'occurrence la plus probable. Les travaux d'Aiyagari, Marcet, Sargent and Seppala (2002) et Shin (2006) montrent, cette fois dans un cadre avec engagement de la puissance publique, que l'hétérogénéité des agents et l'incomplétude des marchés jouent un rôle crucial dans la détermination de la dette publique. Ces travaux sont appuyés par les modèles stationnaires du type Aiyagari et McGrattan (1998) qui suggèrent que des conditions pour garantir une dette positive dans le long terme existent. Ce chapitre avance l'idée que les hypothèses sur l'engagement, le niveau de complétude des marchés et l'incertitude ont un effet important sur le niveau de dette optimal dans le temps.

Chapitre 1

Fluctuations macroéconomiques et dette publique

1 Introduction

Dans ce chapitre de thèse, nous cherchons à analyser l'impact des fluctuations macroéconomiques sur le niveau de dette optimal dans une économie à marchés incomplets. La plupart des travaux antérieurs qui ont cherché à évaluer l'impact de la dette publique indiquent que cette dernière est neutre (Barro (1979)) ou que son introduction n'est pas désirable en raison de l'éviction du capital physique qu'elle induit (Bernheim (1989)). Cependant, ces analyses s'inscrivent dans le cadre de modèles où les marchés sont complets. Woodford (1990) puis Aiyagari et McGrattan (1998) montrent que l'incomplétude des marchés et l'existence d'une contrainte d'endettement à l'origine d'une épargne de précaution réaffirme le rôle de la dette publique. Parce qu'elle permet aux ménages de réduire le coût de l'épargne de précaution, et ainsi d'assurer un meilleur lissage de la consommation, l'introduction de la dette est bénéfique dans ce type d'économies. Cependant, ces analyses ignorent une source non négligeable d'accentuation du risque idiosyncrasique de revenu non assurable qu'est la présence de fluctuations macroéconomiques. L'introduction du risque non assurable dans la littérature sur le coût des fluctuations a conduit à reconsidérer les conclusions de Lucas (1987). Krusell et Smith (2002) ou Storesletten, Telmer et Yaron (2001) trouvent, dans des modèles où les fluctuations macroéconomiques sont corrélées au risque idiosyncrasique, que le coût des fluctuations pourrait être plus élevé que ce que rapporte Lucas (1987). Ce coût des fluctuations pourrait, comme le suggère Imrohoroglu (1989), être réduit par des mesures de politique économique. Le présent chapitre se propose d'explorer cette suggestion du point de vue de la dette publique.

L'objectif principal de ce papier est de proposer un cadre pour évaluer le niveau optimal de dette publique en présence de fluctuations macroéconomiques. Ces fluctuations affectent le comportement d'épargne de précaution des ménages et constituent un facteur important à prendre en compte lors de la mise en place de politiques de dette publique n'ayant pas été considéré par la littérature à ce jour. Nous menons cette analyse dans un modèle calibré pour reproduire les statistiques clés d'une économie de référence. Dans la lignée des modèle de type Bewley-Huggett-

Aiyagari, le modèle présenté dans ce papier introduit une économie avec un système de marchés incomplets où un grand nombre d'agents *ex ante* identiques subissent des chocs de productivité agrégés et des chocs idiosyncratiques. En présence de contraintes de crédit, les agents ont un motif d'épargne de précaution. Le seul moyen d'assurance dont disposent les agents est un actif sans risque composé de capital physique et de titres de dette publique. L'État finance sa dépense publique en émettant de la dette et en prélevant des taxes proportionnelles.

Dans ce cadre, nous trouvons que le ratio dette sur PIB annuel optimal est de 5% pour une économie calibrée sur les États-Unis. Ce niveau est en moyenne plus élevé que dans une économie similaire mais sans fluctuations macroéconomiques, même si en fonction de la manière dont les fluctuations macroéconomiques sont enlevées, l'écart peut être minime. Le mécanisme principal qui explique cet écart est le suivant. Les contraintes de crédit et l'incertitude conduisent les agents à constituer une épargne de précaution. Cette épargne a un coût immédiat en termes de consommation mais permet de lisser celle-ci dans le temps. La présence de fluctuations macroéconomiques exacerbe le niveau de risque dans l'économie et, de ce fait, augmente l'épargne de précaution et écarte le taux d'intérêt du taux de préférence pour le présent. Un *effet fluctuation de l'emploi* modifie le taux et la durée du chômage entre les périodes d'expansion et de récession et affecte le motif d'épargne de précaution. Un *effet fluctuation des prix* modifie de la même façon le niveau des prix et affecte le coût de l'épargne de précaution. Dans ce contexte, un niveau plus élevé de dette publique a deux implications opposées sur l'économie : un *effet d'éviction* qui réduit le bien-être et un *effet coût de l'épargne de précaution* qui améliore le bien-être en réduisant le coût de l'épargne de précaution. Du fait des effets fluctuation de l'emploi et fluctuation des prix, la réduction du coût de l'épargne de précaution par l'*effet liquidité* doit être plus fort dans une économie avec fluctuations macroéconomiques. Ainsi, dans une telle économie le niveau de dette optimal est plus élevé que dans une économie sans fluctuations.

2 Un modèle pour évaluer le niveau de la dette publique en présence de fluctuations macroéconomiques

2.1 Le modèle de référence

Le cadre théorique utilisé ici s'inscrit dans la lignée des travaux de Bewley (1980, 1983), Huggett (1993) et Aiyagari (1994). Les ménages sont exposés à un risque individuel de chômage qui rend incertain la chronique de leurs revenus futurs. Nous introduisons également des fluctuations macroéconomiques et corrélons ces dernières avec les chocs idiosyncrasiques sur le marché du travail. En l'absence de marchés d'assurance complets, les ménages qui subissent une contrainte d'endettement épargnent alors pour un motif de précaution (Aiyagari (1994)).

Dans le secteur productif, à chaque période, le bien consommé par l'agent est produit par une entreprise représentative. Le niveau de production agrégé est donné par la technologie de production suivante[1] :

$$Y_t = z_t F(K_t, N_t) \tag{1.1}$$

où K désigne le stock de capital agrégé et N l'emploi agrégé. Le capital se déprécie au taux constant δ. z représente le progrès technique au sens de Hicks que l'on suppose stochastique. Il prend des valeurs dans l'ensemble $Z = \{z_b, z_g\}$ où z_b et z_g désignent respectivement la valeur de la productivité en récession et en expansion. On suppose que z suit un processus markovien d'ordre un dont les probabilités de transition sont données par $\eta_{zz'} = \Pr(z_{t+1} = z'/z_t = z)$. Par ailleurs, on suppose que le marché des facteurs de production est concurrentiel. Le salaire w et le taux d'intérêt r vérifient :

$$\begin{cases} r_t + \delta = z_t F_K(K_t, N_t) \\ w_t = z_t F_N(K_t, N_t) \end{cases}$$

Le gouvernement doit financer une dépense publique G. Pour ce faire, d'une part, le gouvernement émet de la dette B. D'autre part, il taxe proportionnellement les revenus du capital et du travail des ménages au taux τ. Le niveau agrégé des

[1]La fonction F est à rendements d'échelle constants en K et N, de productivité marginale positive et strictement décroissante et satisfait les conditions d'Inada.

taxes est noté T. La contrainte budgétaire du gouvernement vérifie :

$$G_t + r_t B_t + Tr_t = B_{t+1} - B_t + T_t \tag{1.2}$$

avec

$$T_t = \tau_t \left(w_t N_t + r_t A_t \right)$$

A désigne la richesse totale de l'économie et vérifie $A_t = K_t + B_t$. En outre, l'État procède à un transfert forfaitaire Tr en direction des ménages[2].

Les ménages constituent un continuum d'agents *ex ante* identiques de mesure unitaire dont la durée de vie est infinie. Les préférences d'un agent sont résumées par la fonction V :

$$V = \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \left(\prod_{j=0}^{t} \beta_j u(c_t) \right) \right\}$$

β désigne le facteur d'actualisation que l'on suppose stochastique[3]. Cette hypothèse implique que le taux d'actualisation β diffère entre les agents et varie dans le temps. Ainsi à chaque période, il existe des agents plus patients que d'autres. Cette hypothèse peut être interprétée comme l'existence de dynasties altruistes dont le niveau de patience diffère d'une génération à l'autre. On suppose que le taux d'actualisation suit un processus markovien d'ordre un. c_t désigne la consommation du ménage à

[2]L'existence de fluctuations macroéconomiques modifie les recettes fiscales du gouvernement. Le transfert Tr sert à ajuster à la marge l'effet du cycle sur la contrainte budgétaire de l'Etat. Ce transfert est nul à l'équilibre du modèle.

[3]Cette méthode, proposée par Krusell et Smith (1998), permet de rapprocher la distribution des richesses de sa contrepartie empirique.

la date t. La fonction d'utilité u est continue, strictement concave et de type logarithmique tel que $u(c) = \ln(c)$.

Les agents sont exposés à un risque idiosyncrasique de chômage. On note s le statut de l'agent sur le marché du travail. L'agent est soit au chômage ($s = u$) soit occupe un emploi ($s = e$). L'existence de chocs agrégés exacerbe le risque idiosyncrasique de telle sorte que ce risque est plus élevé en récession et plus faible en expansion. De ce fait, les transitions sur le marché du travail sont corrélées à l'état agrégé de l'économie. On nomme Π la matrice de transition, qui décrit conjointement la transition d'un état individuel et agrégé (z, s) à un autre (z', s').

L'agent employé offre à l'entreprise une unité de travail de façon inélastique. En contrepartie, il reçoit le salaire w. Lorsque l'agent est au chômage, son revenu est noté θ et peut être interprété comme sa production domestique. Les marchés étant incomplets, les ménages ne peuvent qu'imparfaitement s'assurer contre le risque de chômage. En outre, à l'instar de Aiyagari et McGrattan (1998), nous supposons qu'ils ne peuvent s'endetter. Le seul moyen d'assurance dont dispose les ménages est l'accumulation de capital privé ou de créances sur l'État. Cette détention d'actifs rapporte le même intérêt r et est notée a. En notant $v(a, s, \beta; \Gamma, z)$ la fonction valeur optimale d'un agent, le problème que celui-ci doit résoudre peut s'écrire sous la forme récursive suivante :

$$v(a,s,\beta;\Gamma,z)=\max_{c,\,a'}\{u(c)+\beta E\left[v(a',s',\beta';z',\Gamma')|(s,\beta;z,\Gamma)\right]\} \qquad (1.3)$$

sous les contraintes :

$$c+a'=(1+r(z,\Gamma)(1-\tau))a+w(z,\Gamma)\chi(s)+Tr$$

$$c\geq 0$$

$$a'\geq 0$$

$$\Gamma'=H(\Gamma,z,z')$$

avec :

$$\chi\left(s\right)=\begin{cases}\theta \text{ si } s=u,\\ \\ (1-\tau) \text{ si } s=e\end{cases}$$

L'existence de fluctuations macroéconomiques conduit à distinguer les variables d'état individuelles et agrégées. Les variables d'état individuelles sont données par le vecteur (a,s,β). Les variables d'état agrégées sont résumées par le vecteur (z,Γ) où $\Gamma(a,s,\beta)$ est une mesure de la distribution des agents sur la détention d'actifs, l'impatience et le statut sur le marché du travail. Pour prédire le prix des facteurs, les agents doivent être en mesure d'apprécier le capital agrégé et donc la distribution des richesses de la période courante. Les prix courants dépendent donc de la distribution des richesses et du choc agrégé.

L'algorithme qui permet de résoudre ce modèle est présenté en détail dans l'annexe technique du chapitre 2 de cette thèse.

2.2 Définition de l'équilibre

L'équilibre récursif de cette économie consiste à déterminer les règles de décision de consommation et d'épargne $\{c(a,\epsilon,\beta;z,\Gamma), a'(a,\epsilon,\beta;z,\Gamma)\}$, le capital agrégé de l'économie $K(z,\Gamma)$, le prix des facteurs $\{r(z,\Gamma), w(z,\Gamma)\}$, le taux d'imposition τ et la loi d'évolution de la distribution des richesses $\Gamma' = H(\Gamma, z)$ de telle sorte que les conditions suivantes sont satisfaites : (i) les règles de décision $c(a,\epsilon,\widetilde{\beta};z,\Gamma)$ et $a'(a,\epsilon,\widetilde{\beta};z,\Gamma)$ sont solutions du programme de maximisation (1.3) de l 'agent ; (ii) les prix vérifient : $r(z,\Gamma) = zF_K(K,N) - \delta$ et $w(z,\Gamma) = zF_N(K,N)$; (iii) la contrainte budgétaire de l'État est vérifiée ; (iv) l'équilibre sur le marché du capital est vérifié : $K + B = \int a'(a,\epsilon,\beta;\Gamma,z)d\Gamma$; (v) le programme de l'agent est satisfait étant donnée la loi d'évolution H et la loi d'évolution est compatible avec le comportement individuel.

2.3 Étalonnage

Le modèle est étalonné sur l'économie américaine. Une période dans le modèle correspond à un trimestre dans l'économie de référence. La fonction de production est de type Cobb-Douglas :

$$Y_t = z_t F(K_t, N_t) = z_t K_t^{\alpha} N_t^{1-\alpha} \quad 0 < \alpha < 1$$

Le taux de dépréciation δ est fixé à $0,025$. La part du capital dans le produit α est fixée à $0,36$. On suppose, à l'image de Krusell et Smith (1998), que la valeur du choc de productivité agrégé vaut $z_b = 0,99$ lorsque le choc est de type récessionniste et $z_g = 1,01$ lorsque le choc est de type expansionniste. L'utilité des ménages est logarithmique. Afin de reproduire la distribution et l'indice de Gini de la richesse américaine, ainsi que le ratio capital/PIB, nous employons les stratégies qui suivent. Pour générer un grand groupe d'agents pauvres, nous supposons que les chômeurs reçoivent des revenus. Ainsi, le ratio θ est fixé à $0,10$. Pour générer une queue de distribution à droite qui soit épaisse, nous utilisons la méthode des facteurs d'actualisation stochastiques, proposée par Krusell et Smith (1998). Le facteur d'actualisation β peut ainsi prendre trois valeurs $\{\beta_l, \beta_m, \beta_h\}$ où $\beta_l < \beta_m < \beta_h$:

$$\begin{pmatrix} \beta_l \\ \beta_m \\ \beta_h \end{pmatrix} = \begin{pmatrix} 0.9750 \\ 0.9880 \\ 0.9985 \end{pmatrix}$$

On note Υ la matrice de transition qui décrit les transitions d'un facteur d'actualisation à un autre :

$$\Upsilon = \begin{pmatrix} 0,9950 & 0,005 & 0,0000 \\ 0,0007 & 0,9979 & 0,0014 \\ 0,0000 & 0,0050 & 0,9950 \end{pmatrix}$$

Cet étalonnage nous permet de reproduire un Gini de $0,82$, un ratio capital/PIB en équivalent annuel de $2,65$ et les caractéristiques de la distribution des richesses reportées dans le tableau 1.1.

Table 1.1: Caractéristiques de la distribution des richesses

	Haut de la distribution				Gini
Source	1%	5%	10%	20%	
Modèle	22	52	71	89	.82
Données	30	51	64	79	.79
	Bas de la distribution				
Source	20%	40%	60%	80%	
Modèle	1	2	4	11	
Données	0	1	6	18	

L'étalonnage du choc de productivité agrégé et des caractéristiques du marché du travail reprend celui de Krusell et Smith (1998). On suppose que la durée d'un cycle (récession ou expansion) est égale à 8 trimestres. On suppose que la durée moyenne du chômage s'élève en expansion (*resp.* en récession) à $1,5$ trimestres (*resp.* $2,5$). Le taux de chômage vaut en expansion (*resp.* récession) 4% (*resp.* 10%). Ces différentes hypothèses nous permettent de définir la matrice Π qui décrit la transition conjointe d'un état individuel et agrégé vers un autre :

$$\Pi = \begin{pmatrix} 0,5250 & 0,3500 & 0,0313 & 0,0938 \\ 0,0388 & 0,8361 & 0,0021 & 0,1229 \\ 0,0938 & 0,0313 & 0,2916 & 0,5884 \\ 0,0911 & 0,1158 & 0,0243 & 0,8507 \end{pmatrix}$$

Les étapes détaillant l'étalonnage des matrices de transition ci-dessus ainsi que le respect des degrés de liberté lors du processus d'étalonnage est expliqué en détail dans le corps de la thèse ainsi que dans les annexes.

Enfin, à l'image de Aiyagari et McGrattan (1998), on suppose que le ratio dépenses publiques sur PIB est fixé à $0,217$. Le ratio dette/PIB, noté b, vaut $\frac{8}{3}$ dans l'économie de référence, ce qui correspond à une valeur annuelle de $\frac{2}{3}$.

3 L'impact des fluctuations macroéconomiques sur le niveau de dette publique

3.1 Propriétés du modèle de référence

La Figure 1.1 présente les propriétés macroéconomiques du modèle de référence. La hausse du ratio dette/PIB augmente l'offre d'actifs sans risque dans l'économie. Le taux d'intérêt s'ajuste à la hausse et on observe un effet d'éviction standard de la dette publique sur la sphère privée. Cependant puisque le taux d'intérêt augmente, le coût de l'épargne de précaution baisse et les ménages détiennent plus d'actifs à l'équilibre. La hausse du ratio dette/PIB a également un effet mécanique d'augmentation des taxes qui reste contenu par une augmentation simultanée de l'assiette de taxation liée à la hausse des actifs détenus dans l'économie.

En envisageant différents niveaux de dette publique autre que celui de référence, ce modèle permet d'établir un niveau de dette optimal pour cette économie. Nous définissons le niveau de dette publique optimal comme le ratio dette/PIB qui maximise le critère de bien-être utilitariste au sens de Lucas (1987). Ce critère mesure la quantité de consommation qu'il faudrait donner ou retirer à un agent pour le rendre indifférent entre le niveau de dette de référence et un autre niveau. Le calcul de ce critère en présence de facteurs d'actualisation stochastiques est expliqué dans le corps de la thèse et s'inspire de Mukoyama et Sahin (2006).

L'introduction de la dette a deux effets opposés sur le niveau de bien-être de l'économie et l'impact final est *a priori* indéterminé. D'une part, l'*effet d'éviction* a une conséquence négative sur le bien-être puisque le capital privé, puis la consommation s'en trouvent réduits. D'autre part, la hausse du taux d'intérêt provoque un *effet coût de l'épargne de précaution* : le coût de l'épargne de précaution se réduit et le lissage de la consommation dans le temps s'en trouve facilité. Cet effet améliore

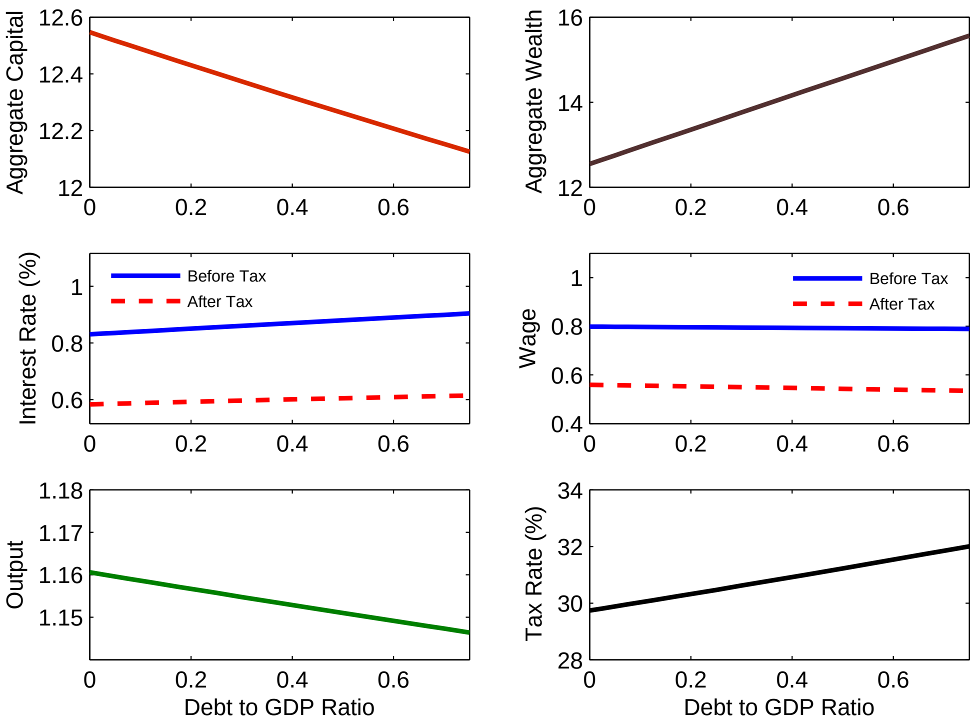

Figure 1.1: Comportement agrégé du modèle de référence

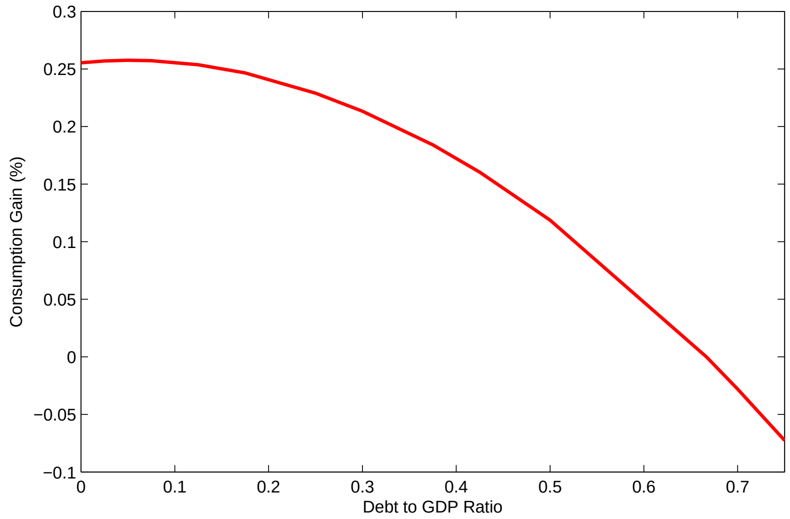

Figure 1.2: Gain de bien-être en fonction du ratio dette/PIB dans l'économie de référence

le bien-être dans l'économie. La figure 1.2 illustre le gain en termes de consommation de différents niveaux de dette publique. Dans une économie prenant en compte les fluctuations macroéconomiques et calibrée sur les Etats-Unis, le ratio dette/PIB optimal est de 5%. Pour ce niveau, l'*effet coût de l'épargne de précaution* compense précisément l'*effet d'éviction*. Pour tout autre niveau, un des effets l'emporte sur l'autre et il est possible d'améliorer le bien-être. La présence de fluctuations macroéconomiques, de risque idiosyncrasique et de contraintes de crédit conduisent les agents à accumuler de l'épargne de précaution. Le niveau d'actif augmente dans l'économie, le taux d'intérêt baisse et s'écarte du taux de préférence pour le présent. Dans le cas présent, un niveau de dette inférieur au niveau de référence améliore le bien-être de l'économie en abaissant le coût de l'effet d'éviction. Dans le même temps, cela renchérit relativement le coût de l'épargne de précaution et a un impact négatif sur le bien-être. Tant que le premier effet l'emporte sur le second, il est possible d'améliorer le bien-être total en abaissant le niveau de dette. Le niveau de dette optimal est atteint quand les deux effets se compensent exactement.

Table 1.2: Gain en consommation (%) à être au niveau optimal au lieu du niveau de référence

	Business Cycle		
Population	Average	Recessions	Booms
All	0.257	0.267	0.247
Bottom 10%	0.885	0.901	0.868
Top 10%	-10.401	-10.535	-10.264

L'impact des fluctuations macroéconomiques est important pour comprendre le niveau de dette optimal. Nos simulations montrent qu'en récession, les gains de bien-être sont plus importants qu'en moyenne ou qu'en expansion. Ceci est illustré par la tableau 1.2. Ceci est dû à deux effets complémentaires liés aux fluctuations macroéconomiques. D'une part, un *effet fluctuation des prix* modifie le coût de l'épargne de précaution dans le cycle : en récession le coût de l'épargne de précaution est plus élevé à cause de taux d'intérêts plus faibles. D'autre part, un *effet fluctuation de l'emploi* augmente la nécessité de l'épargne de précaution dans le cycle : la durée et le taux de chômage varient suivant que l'on est en récession ou en expansion. En récession, il est d'une part plus facile de perdre son emploi si on est employé et d'autre part plus difficile de trouver un emploi si on est chômeur. Ceci conduit les agents à plus d'épargne de précaution. Au final, en récession, la nécessité de l'épargne de précaution augmente en même temps que son coût. Ainsi, accentuer l'*effet fluctuation des prix* ou l'*effet fluctuation de l'emploi* conduit immédiatement à un niveau de dette optimal plus élevé.

3.2 Niveau optimal de dette publique en l'absence de fluctuations macroéconomiques

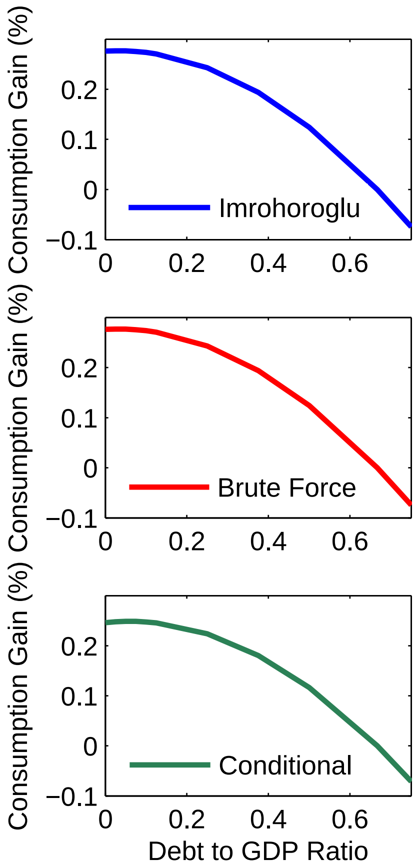

Figure 1.3: Gain de bien-être en fonction du ratio dette/PIB dans l'économie sans fluctuations macroéconomiques

Dans un souci de comparaison, nous caractérisons dans cette section l'impact de la dette dans un environnement économique dépourvu de fluctuations macroéconomiques. Il existe différentes façons d'éliminer les fluctuations macroéconomiques[4]. Nous explorons plusieurs approches. Dans un premier temps, conformément à Lucas (1987), nous fixons la valeur du choc de productivité agrégé à sa valeur moyenne. Nous considérons ensuite trois approches pour obtenir les probabilités de transition sur le marché du travail. La première méthode s'inspire de Imrohoroglu (1989) : les probabilités de transition sur le marché du travail sont fixées afin que le taux de chômage et la durée du chômage en l'absence de fluctuations macroéconomiques correspondent au taux de chômage moyen et à la durée moyenne du chômage dans l'économie de référence avec fluctuations. L'étalonnage des autres paramètres est inchangé. La figure 1.3 (haut) montre que le niveau de dette optimal avec cette méthode est de 2.5%. Ce niveau est inférieur à celui trouvé dans le modèle de référence. Ceci est expliqué par le fait qu'en l'absence de fluctuations, il n'existe pas d'*effet fluctuation des prix* ou d'*effet fluctuation de l'emploi*. En conséquence, l'épargne de précaution est plus faible dans cette économie. Le taux d'intérêt est plus élevé en moyenne et l'effet liquidité est contrebalancé par l'effet d'éviction pour un niveau de dette plus faible.

Dans la seconde méthode, les probabilités de transition sur le marché du travail sont obtenues en prenant la moyenne sur très longue période du processus de transition complet décrit dans le modèle de référence. Cette méthode, qui s'apparente

[4]Se reporter à Barlevy (2004) pour une description des différentes méthodes d'élimination du risque agrégé.

à celle décrite plus haut, permet de trouver des probabilités de transition similaires et un niveau de dette optimal de 2.5% ainsi que le montre la figure 1.3 (milieu). La dernière méthode utilise une approche conditionnelle pour déterminer les probabilités de transition sur le marché du travail. Par exemple, nous calculons la probabilité d'être employé demain sachant que l'on est employé aujourd'hui de la façon suivante :

$$\pi_{ee} = \Pr(z = z_g | s = e) * (\Pi_{bgee} + \Pi_{ggee}) + (1 - \Pr(z = z_g | s = e)) * (\Pi_{gbee} + \Pi_{bbee}),$$

avec par exemple $\Pi_{ggee} = \Pr(z' = z_g, s' = e | z = z_g, s = e)$. Cette méthode conduit à un processus plus persistent sur le marché du travail. En conséquence, le niveau de dette optimal avec cette méthode est plus élevé qu'avec les méthodes ci-dessous et se rapproche du niveau de référence. Ceci est illustré par la figure 1.3 (bas).

4 Conclusion

Ce chapitre montre que la prise en compte des fluctuations agrégées augmente en moyenne le niveau de dette optimal dans l'économie. Ces fluctuations conduisent les agents à accumuler plus d'épargne de précaution. Plus précisément, à la fois le motif et le coût de l'épargne de précaution sont modifiés par un *effet fluctuation de l'emploi* et un *effet fluctuation des prix.* La hausse des actifs détenus dans l'économie baisse le taux d'intérêt et l'éloigne du taux de préférence pour le présent. La hausse du niveau de dette augmente le taux d'intérêt et permet de réduire le coût de l'épargne de précaution et ainsi d'améliorer le bien-être dans l'économie. Ainsi, la présence de dette publique permet en général de réduire le coût des fluctuations pour les agents.

Chapitre 2

Risque entrepreneurial et dette publique

1 Introduction

Dans ce chapitre nous évaluons l'impact de la prise en compte du risque entrepreneurial sur le niveau de dette optimal dans une économie à marchés incomplets. L'article d'Aiyagari et McGrattan (1998) met en évidence qu'il peut être optimal d'avoir un ratio dette sur PIB positif pour l'économie. Cependant, ce résultat s'appuie sur le comportement d'épargne de précaution des agents dans une économie où seul l'emploi salarié est autorisé. Les résultats empiriques montrent qu'une fraction importante de l'épargne nationale est réalisée par les ménages entrepreneuriaux. Ces agents constituent une petite fraction de l'économie mais perçoivent aux Etats-

Unis 22% du revenu national et possèdent 40% de la richesse totale. Par ailleurs, ces agents ont un portefeuille très peu diversifié avec une forte composante investie dans leur entreprise. Les travaux de Gentry et Hubbard (2004) et Covas (2006) montrent que face au risque subi par le entrepreneurs, ceux-ci utilisent les autres actifs dont ils disposent comme moyen d'assurance et de ce fait constituent une épargne de précaution. Dans ce chapitre nous montrons que la prise en compte d'agents entrepreneuriaux est une étape importante pour comprendre les effets de la dette publique dans une économie à marchés incomplets. La prise en compte de l'emploi salarié seul peut mal estimer le niveau de dette optimal et la réponse des entrepreneurs aux variations de la dette peut être très différents des agents employés. Enfin, il est important dans la discussion sur le niveau de dette optimal d'avoir une bonne reproduction de la distribution des richesses de l'économie. En effet, comme le suggère Ball et Mankiw (1995) ou Floden (2001), les gains et les pertes liés aux variations de la dette publique sont réparties différemment selon la richesse d'un agent. Les modèles avec une composante entrepreneuriale permettent de reproduire correctement la distribution des richesses.

Ce chapitre introduit une économie comportant à la fois des agents salariés et des agents entrepreneuriaux subissant un risque idiosyncrasique non assurable. L'incomplétude des marchés et la présence de contraintes de crédit sont également considérées. Nous montrons que le niveau de dette optimal est négatif dans un tel cadre. En effet, dans ce modèle, les ménages salariés ne sont pas à l'origine de l'ensemble de l'épargne dans l'économie puisque les ménages entrepreneuriaux en assurent une grande partie. Tout se passe comme si une partie de l'accumulation réalisée par les ménages salariés dans un modèle à la Aiyagari et McGrattan (1998) se déportait vers les ménages entrepreneuriaux. De ce fait, ce modèle se rapproche des faits empiriques du point de vue des comportements d'accumulation. Ce chapitre argumente qu'en présence de ménages entrepreneuriaux, les ménages salariés sont plus sensibles aux effets néfastes de l'éviction du capital privé suite à l'augmentation du niveau de dette publique. Les ménages salariés ne souhaitent donc pas un niveau de dette publique positif. Au contraire, les ménages entrepreneuriaux qui subissent un risque idiosyncrasique non assurable important souhaitent se prémunir contre le risque en constituant une épargne de précaution. De ce fait, ces agents tendent à préférer un niveau de dette publique positif. Au final, du fait que les ménages salariés constituent une fraction plus importante de la population, leur comportement ressort au niveau agrégé et le niveau de dette optimal est négatif.

2 Un modèle pour évaluer l'impact de la dette publique dans une économie entrepreneuriale

2.1 Le modèle de référence

Le cadre théorique utilisé ici s'inscrit dans la lignée des travaux de Bewley (1980, 1983), Huggett (1993) et Aiyagari (1994). Les ménages sont exposés à un risque individuel non totalement assurable. Les ménages salariés subissent un risque sur le marché du travail et les ménages entrepreneuriaux un risque sur leur activité de production. Nous supposons qu'une fraction fixe de la population s'engage dans une activité entrepreneuriale et que le reste de la population offre son travail de manière élastique à un secteur de production représentatif.

Dans cette économie, un bien unique est produit par deux secteurs de production. Dans le secteur entrepreneurial, la production est réalisée grâce à la technologie suivante :

$$Y_t^e = \theta_t f(k_t)$$

avec f une fonction à rendement d'échelle décroissant, k le capital placé dans l'activité risquée par l'entrepreneur et θ la productivité. La productivité est variable et suit un processus markovien d'ordre un. Le secteur représentatif opère avec une technologie à rendement d'échelle décroissant :

$$Y_t^c = F(K_t, Z_t L_t).$$

où K est le capital agrégé et L est l'offre de travail agrégé stationnarisé. Z est la productivité du travail. Z croît au taux exogène g de sorte que $Z_t = (1+g)^t$. Le capital se déprécie dans les deux secteurs au même taux δ.

Le programme récursif des ménages entrepreneuriaux est le suivant :

$$v^e(\theta, x) = \max_{c,k',a^{e\prime}} U^e(c) + \beta\mathbb{E}[v^{e\prime}(\theta', x')|\theta] \tag{2.1}$$

sous la contrainte de

$$c + k' + a^{e\prime} = x$$

$$x' = \theta' f(k') + (1-\delta)k' + Tr + (1+r)a^{e\prime} - \tau\xi'$$

$$\xi = \theta f(k) - \delta k + ra^e$$

$$k' \geq 0 \textit{ et } a^{e\prime} \geq \bar{a}^e$$

La contrainte budgétaire indique que ces agents répartissent leur épargne entre un actif certain a^e rémunéré au taux r et un actif risqué k. c désigne leur consommation, Tr un transfert de l'état et τ un taux de taxe. v^e est la fonction valeur optimale de ces ménages, x leur richesse courante, U^e une fonction d'utilité et β le taux d'escompte.

Le programme récursif des ménages salariés est le suivant :

$$v^{ne}(e, y) = \max_{c,l,a^{ne\prime}} U^{ne}(c, l) + \beta\mathbb{E}[v^{ne\prime}(e', y')|e] \tag{2.2}$$

sous la contrainte de

$$c + a^{ne\prime} = y$$

$$y' = (1-\tau)we'l' + Tr + (1 + r(1-\tau))a^{ne\prime}$$

$$a^{ne\prime} \geq \bar{a}^{ne}$$

La contrainte budgétaire indique que ces agents répartissent leur épargne dans un actif unique certain a^{ne} rémunéré au taux r. c désigne leur consommation, Tr un transfert de l'état et τ un taux de taxe. Ces ménages offrent la quantité de travail l rémunéré au salaire w. Ils subissent un risque de revenu sur le marché du travail noté e qui suit un processus markovien d'ordre un. v^{ne} est la fonction valeur optimale de ces ménages, y leur richesse courante, U^{ne} une fonction d'utilité et β le taux d'escompte.

Le gouvernement émet de la dette publique et collecte une taxe proportionnelle afin de financer la dépense publique. La contrainte budgétaire du gouvernement s'écrit :

$$G + Tr + rB = B' - B + T$$

avec G la dépense publique, Tr un transfert forfaitaire, B le niveau de la dette publique et T le montant des impôts collectés. Par hypothèse, la dépense publique, le transfert, et le niveau de la dette sont fixés de façon à représenter une fraction constante du PIB. De ce fait, le taux de taxe à l'état stationnaire s'écrit :

$$\tau = \frac{\gamma + (r-g)b + \varphi}{1 - \delta\bar{k} + rb} \tag{2.3}$$

où γ est le ratio de la dépense publique rapportée au PIB, b est le ratio dette sur PIB, φ le ratio de transfert rapporté au PIB. $\bar{k}$ est le ratio du capital rapporté au PIB. Nous considérons l'équilibre de long terme stationnaire du modèle où les variables appropriées ont été amorties des effets de la croissance.

2.2 Définition de l'équilibre

L'équilibre récursif stationnarisé[1] de cette économie consiste à déterminer une fonction valeur $\hat{v}^e(\theta, \hat{x})$ pour le ménage entrepreneurial ; et $\hat{v}^{ne}(e, \hat{y})$ pour le ménage salarié ; les règles de décision $\hat{k}(\theta, \hat{x}), \hat{a}^e(\theta, \hat{x}), \hat{c}^e(\theta, \hat{x})$ pour le ménage entrepreneurial ; et $\hat{a}^{ne}(e, \hat{y}), \hat{c}^{ne}(e, \hat{y})$ pour le ménage salarié ; les prix des facteurs $(r, \hat{w})$; la demande de capital et de travail du secteur représentatif $(\hat{K}, L)$; une distribution constante des caractéristiques entrepreneuriales $\Gamma^e(\theta, \hat{x})$ de masse χ ; une distribution constante des caractéristiques salariales $\Gamma^{ne}(e, \hat{y})$ de masse $(1 - \chi)$; un taux de taxe τ, de telle sorte que les conditions suivantes sont satisfaites : (i) étant donné r et τ, les règles de décision du ménage entrepreneurial résolvent le programme de celui-ci, (ii) étant donné r, $\hat{w}$ et τ, les règles de décision du ménage salarié résolvent le programme de celui-ci, (iii) le capital agrégé $\hat{K}$, l'emploi agrégé L et le capital entrepreneurial agrégé $\hat{K}^e$ sont donnés par : $\hat{K} + \hat{B} = \sum_{\theta \in \Theta} \int \hat{a}^e(\theta, \hat{x}) d\Gamma^e(\theta, \hat{x}) + \sum_{e \in \mathrm{E}} \int \hat{a}^{ne}(e, \hat{y}) d\Gamma^{ne}(e, \hat{y})$; $L = \sum_{e \in \mathrm{E}} \int el(e, \hat{y}) d\Gamma^{ne}(e, \hat{y})$; $\hat{K}^e = \sum_{\theta \in \Theta} \int \hat{k}(\theta, \hat{x}) d\Gamma^e(\theta, \hat{x})$, (iv) étant donné $\hat{K}$ et L, le prix des facteurs vérifient : $r = F_{\hat{K}}(\hat{K}, ZL) - \delta$; $\hat{w} = F_L(\hat{K}, ZL)$, (v) étant données les règles de décision des ménages entrepreneuriaux et salariés, les mesures de probabilité Γ^e et Γ^{ne} sont

[1] Nous adoptons la convention de noter une variable stationnarisée x d'après cette définition : $\hat{x}_t = \frac{x_t}{(1+g)^t}$.

constantes, (vi) le budget du gouvernement est équilibré.

2.3 Etalonnage

La périodicité du modèle est l'année. Un premier groupe de paramètres (tableau 2.1) est tiré de la littérature empirique. Le second groupe de paramètres (tableau 2.2) est calibré de façon à reproduire les objectifs d'étalonnage suivants : un ratio capital sur PIB de 2.5, le Gini et les quantiles de la distribution de la richesse aux Etats-Unis. L'étalonnage détaillé peut être trouvé dans le corps de la thèse.

Table 2.1: Paramètres étalonnés d'après la littérature empirique

Paramètre	Valeur	Signification
α	0.3	Part du capital dans la production
δ	0.075	Taux de dépréciation
χ	0.0755	Fraction d'entrepreneurs dans l'économie
g	0.0185	Taux de croissance
b	$\frac{2}{3}$	Ratio dette sur PIB de référence
μ	1.5	Aversion au risque
η	0.328	Elasticité de l'offre de travail
$\bar{a}^{ne}$	0	Limite d'endettement du salarié
$\bar{a}^{e}$	-4	Limite d'endettement de l'entrepreneur
ρ^{ne}	0.9	Paramètre autorégressif du processus de revenu salarial
ε^{ne}	0.21	Ecart-type du processus de revenu salarial
γ	0.217	Ratio dépense publique sur PIB
φ	0.082	Ratio transfert sur PIB

2.4 Résultats

La figure 2.1 détaille le comportement agrégé du modèle de référence. L'augmentation du ratio dette sur PIB augmente le taux d'intérêt avant et après taxes, baisse le

Table 2.2: Paramètres étalonnés pour reproduire les objectifs de calibration

Paramètre	Value	Signification
β	0.978	Taux d'escompte
θ_{low}	0.3	Productivité basse de l'activité entrepreneuriale
θ_{high}	1.11	Productivité haute de l'activité entrepreneuriale
π_{11}	0.988	Probabilité de transition (bas vers bas)
π_{21}	0.115	Probabilité de transition (haut vers bas)
ν	0.6	Degré de rendement d'échelles décroissant

salaire et l'offre de travail. Le taux de taxe présente le comportement non monotone décrit par Aiyagari et McGrattan (1998) dû à l'assiette de taxation. L'augmentation du ratio dette sur PIB provoque l'éviction du capital privé employé dans le secteur représentatif.

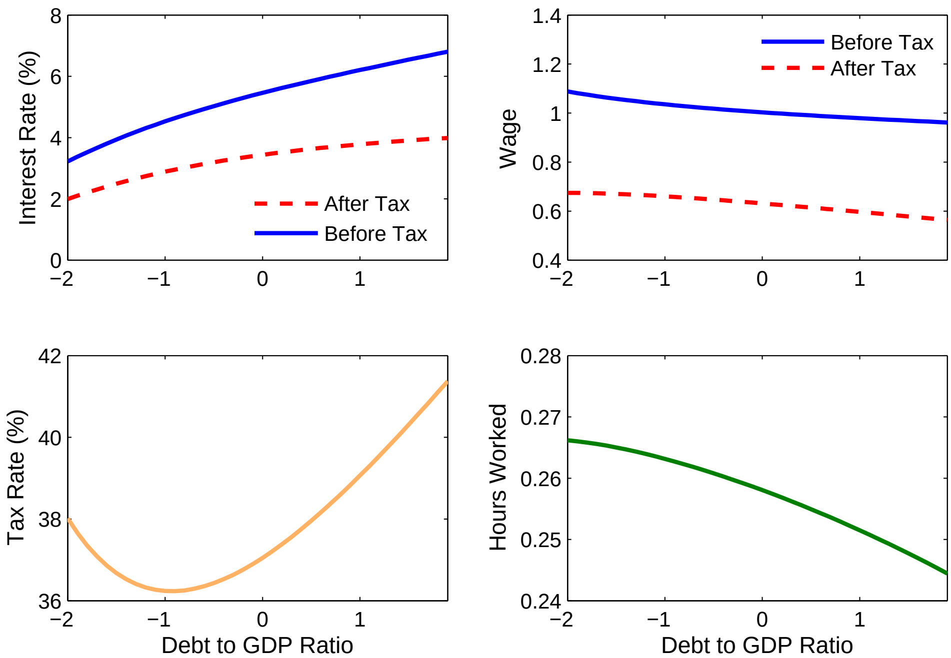

Figure 2.1: Comportement agrégé du modèle de référence

La figure 2.2 rapporte le comportement agrégé des variables relatives aux ménages entrepreneuriaux. L'épargne dans l'actif certain augmente avec le ratio dette sur PIB. Ce résultat est comparable à ceux de Covas (2006) qui souligne qu'en présence de risque partiellement assurable, les entrepreneurs ont tendance à investir massivement dans l'actif sans risque. Toutefois à l'équilibre général, le comportement d'épargne dans cet actif baisse le taux d'intérêt et rend de ce fait l'actif risqué plus attractif. L'augmentation de la dette publique en élevant le taux d'intérêt permet de réduire le coût de l'épargne non risquée et de ce fait celle-ci augmente avec le ratio dette sur PIB. L'accumulation dans l'actif risqué est quant à lui non monotone en fonction du ratio dette sur PIB. Pour des valeurs faibles de ce ratio, au lieu d'être évincée, l'épargne risquée augmente avec la dette. Ceci s'explique par un effet de richesse : tant que le rendement de l'actif risqué est supérieur à celui de l'actif non risqué, les agents souhaitent investir dans cet actif. L'augmentation du taux d'intérêt provenant de l'augmentation de la dette permet aux agents de profiter des revenus plus importants de l'épargne non risquée pour investir à la fois dans l'actif risqué et l'actif non risqué. Lorsque le rendement de l'actif risqué est inférieur au rendement de l'actif non risqué, on observe l'effet d'éviction habituel.

La figure 2.3 (courbe en trait plein) détaille les résultats principaux du modèle de référence. Le ratio optimal de dette obtenue dans un économie où le risque entrepreneurial est pris en compte est de -110%. Les gains en termes de consommation seraient de 1.8% si l'économie était au niveau optimal au lieu du niveau de référence. Ce résultat s'explique de la façon suivante. La propension à épargner des

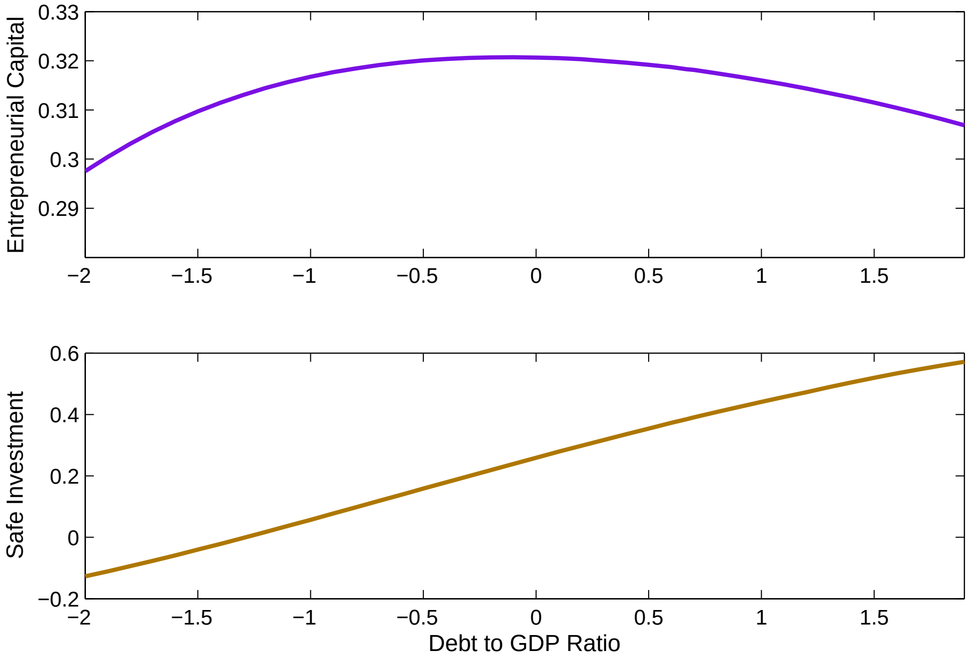

Figure 2.2: Comportement agrégé des variables entrepreneuriales

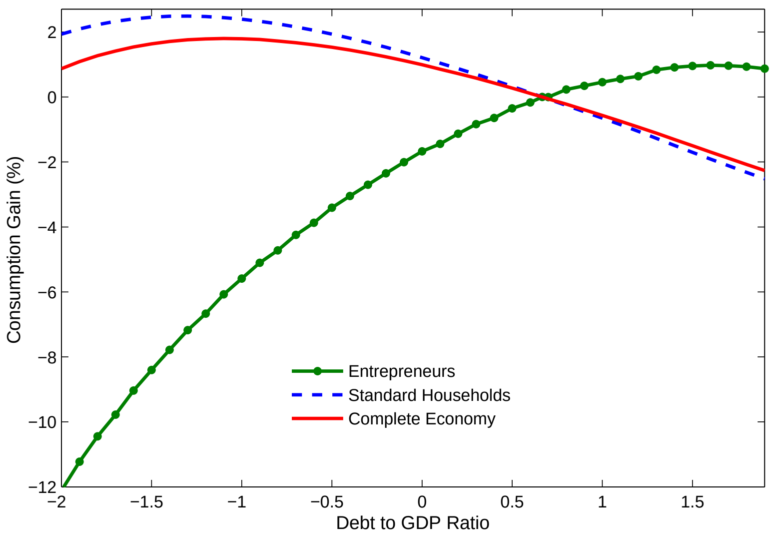

Figure 2.3: Gain en termes de consommation en fonction de la dette

entrepreneurs étant supérieure à celle des salariés, l'introduction de cette catégorie de ménages dans un modèle de type Aiyagari et McGrattan (1998) tend à augmenter le ratio capital sur PIB, toutes choses égales par ailleurs. De ce fait, dans un modèle étalonné avec entrepreneurs, les salariés ont tendance à moins épargner car leur épargne est remplacée par celle plus factuelle des entrepreneurs[2]. Le comportement d'accumulation des ménages salariés étant plus faible ici, ceux-ci sont plus préoccupés par les effets néfastes sur leur bien-être de l'éviction du capital privé et sont moins concernés par l'effet positif de la réduction du coût de l'épargne de précaution. En d'autres termes, les ménages salariés ont tendance à préférer un niveau de dette faible. Ceci est illustré sur la figure 2.3 (courbe pointillée). Cette courbe représente le bien-être pour les ménages salariés seuls et indique que le niveau optimal pour ces derniers est de -130%. Au contraire, le niveau optimal de dette pour les entrepreneurs seuls est positif comme l'indique la figure 2.3 (courbe cerclée) et se fixe à 160%. Ceci vient du fait que les entrepreneurs subissent un risque idiosyncrasique important et qu'un niveau de dette élevé les aide à amortir le coût de l'épargne de précaution. Par ailleurs, en dehors de l'effet sur le niveau optimal de dette provenant de l'ajustement du taux d'escompte, on observe un autre effet de l'introduction de ménages entrepreneuriaux sur le niveau de dette voulu par les ménages salariés. Même en l'absence d'ajustement du facteur d'escompte, l'introduction de ménages entrepreneuriaux tend à augmenter le taux de taxe, ce

[2]D'un point de vue technique, pour garder le ratio K/Y constant dans une économie avec des entrepreneurs, il est nécessaire de diminuer le taux d'escompte, toutes choses égales par ailleurs.

qui décourage les ménages salariés d'opter pour un niveau de dette plus élevé. En effet, toutes choses égales par ailleurs, l'introduction de ménages entrepreneuriaux tend à abaisser le taux d'intérêt de l'économie puisque ces derniers ont un motif d'épargne de précaution plus important que les ménages salariés. Comme le montre l'équation 2.1, l'impact d'une variation du taux d'intérêt sur le taux de taxe est ambigu. L'introduction de ménages entrepreneuriaux tend également à augmenter le ratio capital sur PIB $\bar{k}$. Ceci tend à augmenter le taux de taxe. De ce fait, l'introduction de ménages entrepreneuriaux conduit à une augmentation du taux de taxe car l'effet du ratio capital sur PIB et l'effet du taux d'intérêt allant dans le sens d'une augmentation du taux de taxe domine l'effet du taux d'intérêt allant dans le sens d'une diminution du taux de taxe. Ceci s'ajoute à l'aspect néfaste de l'éviction du capital privé pour les ménages salariés et explique que ceux-ci désirent un niveau de dette moins élevé. Au final, le niveau de dette optimal de l'économie dans son ensemble se détermine en prenant en compte de façon pondérée le niveau optimal pour les ménages entrepreneuriaux et les ménages salariés. Comme ces derniers sont plus nombreux dans l'économie, ils tirent le niveau optimal de dette vers le bas et contribuent à le fixer à -110%.

Table 2.3: Distribution des richesses : comparaison entre les données et les modèles

Modèle	Ratio K/Y	Gini richesse	% de richesse détenue par le haut de la distribution			
			1%	5%	10%	20%
Aiyagari and McGrattan (1998)	2.5	0.42	4	15	26	44
Floden (2001)	2.5	0.63	6	25	41	64
Modèle sans entrepreneurs	2.1	0.43	4	15	27	45
Modèle de référence	2.5	0.78	30.8	55.5	63.5	75.2
Economie américaine	2.5	0.78	29.5	53.5	66.1	79.5

Enfin, comme le montre le tableau 2.3, le niveau optimal de dette trouvé et l'argumentation pour le justifier s'établissent dans un modèle qui assure une bonne reproduction de la distribution des richesses. Il apparaît que le modèle d'Aiyagari et McGrattan (1998), même en introduisant un processus de productivité du travail plus persistant (le tableau 2.3 illustre un tel processus en reprenant l'étalonnage de Floden (2001)) ne permet pas une reproduction même approximative de la distribution des richesses ou du Gini de la richesse. Plusieurs travaux (voir Floden (2001) ou Ball and Mankiw (1995)) discutent de l'importance d'une bonne reproduction de la distribution des richesses pour établir le niveau optimal de la dette publique en raison de l'impact de la richesse d'un ménage sur ce dernier.

3 Conclusion

Ce chapitre montre que l'introduction de ménages entrepreneuriaux dans une économie à marchés incomplets dans l'esprit d'Aiyagari (1994) et Aiyagari et McGrattan (1998) tend à tirer le niveau de dette optimal vers le bas. En effet, les entrepreneurs ayant une propension importante à épargner, une partie de l'accumulation réalisée auparavant par les ménages salariés seuls se trouve transférée. Le chapitre montre que le niveau optimal de dette désirée par les ménages entrepreneuriaux est très différent du niveau désiré par les ménages salariés. Ces derniers souhaitent un niveau de dette négatif car ils sont plus sensibles à l'éviction du capital privé qu'à l'abaissement du coût de l'épargne de précaution accompagnant une hausse de la dette publique. Les ménages entrepreneuriaux, au contraire, ont tendance à vouloir un niveau de dette positif élevé car subissant des risques importants. Il est avantageux pour eux d'abaisser le coût de l'épargne de précaution à travers un niveau de dette plus important.

Chapitre 3

Dette publique optimale sans engagement

1 Introduction

Choisir le niveau de dette optimal dans le temps exige que l'on s'intéresse à la question de l'engagement de la puissance publique. Un pan récent de la littérature, mené par les papiers de Krusell (2002), Klein et Rios-Rull (2002) ou encore Klein, Krusell et Rios-Rull (2007), s'intéresse aux politiques publiques en l'absence de possibilité d'engagement puisqu'il n'existe pas de méthode parfaite pour garantir cet engagement. De telles politiques permettent d'appréhender la cohérence temporelle de la puissance publique et se réfèrent à des gouvernements successifs jouant un jeu contre eux-mêmes dans le temps. Afin de comprendre la raison pour laquelle une majorité d'Etats entretiennent un niveau de dette important, Krusell, Martin et Rios-Rull (2006) reprennent le modèle de Lucas et Stokey (1983) et relâchent l'hypothèse d'engagement. Leurs résultats ne permettent pas de souligner en général qu'un niveau de dette élevé est l'occurrence la plus probable. Les travaux de Aiyagari, Marcet, Sargent and Seppala (2002) et Shin (2006) montrent, cette fois dans un cadre avec engagement de la puissance publique, que l'hétérogénéité des agents et l'incomplétude des marchés jouent un rôle crucial dans la détermination de la dette publique. Ces travaux sont appuyés par les modèles stationnaires du type Aiyagari et McGrattan (1998) qui suggèrent que des conditions pour garantir une dette positive dans le long terme existent.

Ce chapitre avance l'idée que les hypothèses sur l'engagement, le niveau de complétude des marchés et l'incertitude ont un effet important sur le niveau de dette optimal dans le temps. L'élément factuel selon lequel une majorité de gouvernements présentent un niveau de dette positif et élevé n'est pas reproduit par les modèles avec engagements. Ce chapitre propose un cadre pour analyser le niveau de dette optimal sans engagement de la puissance publique où la question de l'incomplétude des marchés, de l'incertitude et de la contrainte de crédit est considérée. Le cadre de référence présenté ici s'inscrit dans la tradition des modèles de type Bewley-Huggett-Aiyagari mais introduit la considération de la cohérence temporelle du choix de la dette publique par la puissance publique. Les agents subissent un risque idiosyncrasique sur le marché du travail qu'ils ne peuvent parfaitement assurer du fait de l'incomplétude des marchés et sont contraints sur le crédit. Une firme représentative produit un bien unique. Le gouvernement finance sa dépense publique en levant une taxe proportionnelle et émet des titres de dette publique. Le gouvernement est également soumis à un choix politique : à chaque période, il doit choisir avec une certaine probabilité le niveau de dette publique pour la période suivante. Dans le cas où un choix doit effectivement être fait, la puissance publique maximise un critère de bien-être social de façon à déterminer le niveau de dette optimal pour la période suivante. Sinon, le gouvernement reconduit le niveau de dette courant. Nous trouvons que le niveau optimal de dette publique trouvé par le modèle de référence est positif et élevé comparé à un modèle stationnaire. Ce résultat s'explique de la façon suivante. Dans un équilibre avec cohérence temporelle, les agents sont rationnels et peuvent prédire le résultat du processus de choix de dette future. Ainsi, ils décident de leur propre choix en sachant que ce choix pourra influencer la dynamique de l'économie. Choisir d'augmenter le niveau de dette aura pour effet immédiat d'abaisser le taux de taxe. Cependant, les agents anticipent que ce taux devra augmenter dans les périodes suivantes. De même, augmenter le niveau de dette aujourd'hui élève le taux d'intérêt immédiatement mais celui-ci aura tendance à s'ajuster à la baisse dans le futur. Un taux d'intérêt plus élevé facilite l'épargne de précaution des agents et a un effet bénéfique sur le bien-être. Toutefois, ceci évince également le capital privé et a un effet néfaste sur le bien-être. Les agents arbitreront entre les gains immédiats et les pertes à terme afin de fixer leur choix. Par la suite, la puissance publique déterminera le niveau de dette qui maximise le bien-être de l'économie.

2 Un modèle pour déterminer le niveau de dette optimal avec cohérence temporelle de la puissance publique

2.1 Le modèle de référence

Le cadre théorique utilisé ici s'inscrit dans la lignée des travaux de Bewley (1980, 1983), Huggett (1993) et Aiyagari (1994) mais étend également le travail de Krusell, Quadrini et Rios-Rull (1997). Ce cadre théorique peut se décrire en deux étapes : (i) une loi d'évolution de l'économie, (ii) une règle de choix de la dette publique. La loi d'évolution détermine le niveau de capital futur de l'économie étant donné l'état actuel. La règle de choix détermine le niveau de dette publique choisie par la puissance publique en fonction de l'état de l'économie.

La production est assurée par une firme représentative utilisant la technologie à rendements constants suivante :

$$Y = F(K, N)$$

avec K est le capital agrégé et N l'emploi agrégé. Le capital se déprécie au taux δ. L'Etat émet des titres de dette publique et taxe les revenus des agents à un taux proportionnel τ pour financer la dépense publique. A chaque date, le gouvernement peut ajuster le niveau de la dette. La contrainte budgétaire du gouvernement s'écrit :

$$G + rB + TR = B' - B + T$$

avec

$$T = \tau(wN + rA)$$

G désigne la dépense publique, B le niveau de la dette publique[1], T le montant de l'impôt et TR des transferts forfaitaires. A désigne la richesse totale dans l'économie de sorte que $A = K + B$. Le taux de taxe peut être déterminé de la façon suivante :

$$\tau = \frac{G + (1+r)B - B' + TR}{(wN + rA)} \tag{3.1}$$

L'économie est composée d'un continuum de ménages de mesure un, ex ante identiques et dont les préférences sont exprimées par la fonction V suivante :

$$V = \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t u(c_t) \right\} \tag{3.2}$$

[1]La dette courante désigne le niveau de dette remboursée dans la période et non le montant émis. De ce fait, le niveau de dette B est une variable prédéterminée pour la période courante alors que B' désigne le choix du niveau de dette à émettre et qui sera remboursée à la période suivante. Les ratios de dépense publique et de transferts sont constants.

β désigne le taux d'escompte et u une fonction d'utilité. Nous supposons que l'offre de travail est inélastique. A chaque période, les ménages sont soumis à un choc s de revenu sur le marché du travail non entièrement assurable. La dynamique de ce choc est donnée par un processus markovien d'ordre un. De plus, une contrainte de crédit empêche les agents de s'endetter indéfiniment de sorte que leur niveau d'actifs a doit respecter la contrainte $a' \geq a_{min}$, avec a_{min}, le niveau minimal d'actifs pouvant être détenu par un agent. Les décisions d'un ménage type dépendent donc de l'état individuel de l'agent mais également de l'état agrégé de l'économie de sorte que l'espace d'état s'écrit (a, s, Ψ, B, B') où Ψ est la fonction de densité des ménages sur l'espace d'état. Cependant, nous devons également définir la façon dont les ménages anticipent les prix et le taux de taxe futurs. Les prix futurs dépendent du niveau de capital futur anticipé, de ce fait, il est nécessaire de fournir une loi d'évolution de la distribution des agents $\Gamma(\Psi, B, B')$. Cependant, comme cette distribution est un objet dans un ensemble de dimension infinie, nous nous appuyons sur la méthodologie introduite par Den Haan (1996) et Krusell and Smith (1998) : la distribution est approximée par le niveau moyen des actifs détenus. De ce fait, nous notons la loi d'évolution du niveau moyen des actifs détenus comme suit : $\hat{\Gamma}(A, B, B')$. Le taux de taxe quant à lui dépend du niveau de dette future, il est donc nécessaire de définir une règle de choix de la dette que l'on note $\Theta(\Psi, B)$ puis $\hat{\Theta}(A, B)$ une fois que la distribution a été approximée. Le programme récursif des agents est donc le suivant :

$$V(a,s,A,B,B') = \max_{c,a'} u(c) + \beta EV(a',s',A',B',B'') \tag{3.3}$$

s.t.

$$a' = (1 + r(1 - \hat{\tau}(A,B,B')))a + ws(1 - \hat{\tau}(A,B,B')) + TR$$

$$A' = \hat{\Gamma}(A,B,B')$$

$$B'' = \hat{\Theta}(A',B') \text{ with probability } \lambda$$

$$B'' = B' \text{ with probability } (1-\lambda)$$

$$r = \hat{r}(A)$$

$$w = \hat{w}(A)$$

$$c \geq 0$$

$$a' \geq a_{min}$$

avec r le taux d'intérêt, w le salaire et λ la probabilité qu'un choix de dette ait lieu à la période courante. Dans ce programme, B' apparaît comme une variable d'état car nous autorisons des variations de la dette future. De ce fait, ce programme n'indique pas comment le niveau de dette pour la période future est choisi à la période courante. Ce choix est justement le jeu dynamique de la puissance publique et est décrit comme suit. A chaque date, nous supposons que la puissance publique peut choisir un niveau de dette avec une certaine probabilité λ. Dans le cas limite où $\lambda = 1$, le gouvernement choisit un niveau de dette à chaque période. Même si le gouvernement est libre de choisir le niveau de dette à la période courante, cette politique économique ne dure que jusqu'au prochain choix de dette puisqu'il n'existe pas de technologie pour assurer un engagement de la puissance publique. Comme le gouvernement n'a pas de prise sur les choix futurs, il est possible de considérer que celui-ci joue un jeu avec les futurs choix comme fonction de réaction. Au final, la puissance publique fixe le niveau de dette pour la période suivante en maximisant le critère utilitariste suivant :

$$B' = \max_{B'} \int_{a_{min}}^{a_{max}} V\left(a, s, A, B, B'\right) \Psi(a) da$$

2.2 Equilibre

L'équilibre récursif est caractérisé par le vecteur $[a'(a, s, A, B, B), V(a, s, A, B, B'), \hat{\Gamma}(A, B, B'),$ $\hat{\Theta}(A, B)]$ de sorte que : (i) étant donnée la loi d'évolution $\hat{\Gamma}\left(A, B, B'\right)$ et la règle de choix $\hat{\Theta}\left(A, B'\right)$, $V\left(a, s, A, B, B'\right)$ est la fonction valeur solution du programme 3.3 et $a'\left(a, s, A, B, B'\right)$ est la règle d'épargne associée, (ii) Etant donnée la fonction valeur ci-dessus, la maximisation du critère de bien-être utilitariste à chaque date est conforme avec la règle de choix anticipée $\hat{\Theta}\left(A, B\right)$ de sorte que $\forall B, \forall A, \hat{\Theta}\left(A, B\right) = \max_{B'} \int_{a_{min}}^{a_{max}} V\left(a, s, A, B, B'\right) \Psi(a) da$, (iii) Etant donnée la règle d'épargne $a'\left(a, s, A, B, B'\right)$ et pour tout état de l'économie (A, B), le niveau moyen des actifs détenus à la période suivante est défini par $A' = \sum_{s \in S} \int_{a_{min}}^{a_{max}} a'\left(a, s, A, B, B'\right) \Psi\left(a\right) da$ est cohérent avec la loi d'évolution anticipée $\hat{\Gamma}\left(A, B, B'\right)$.

2.3 Etalonnage

A cause de la dynamique complexe du modèle, notre stratégie d'étalonnage introduit une étape intermédiaire où nous étalonnons un modèle stationnaire avec des caractéristiques similaires au modèle de référence. Ces paramètres sont ensuite utilisés pour étalonner le modèle de référence. La périodicité du modèle est annuelle et notre cible d'étalonnage principal est le ratio capital sur PIB des Etats-Unis. La technologie de production retenue est une fonction Cobb-Douglas et la fonction d'utilité est de type CRRA. Le tableau 3.1 résume les paramètres retenus. Plus de détails sur cet étalonnage peuvent être trouvés dans le corps de la thèse.

Table 3.1: Paramètres d'étalonnage

Paramètre	Valeur	Signification
α	0.3	Part du capital dans la production
δ	0.075	Taux de dépréciation
μ	3	Aversion au risque
a_{min}	0	Contrainte de crédit
ρ	0.95	Paramètre autorégressif du processus de revenu salarial
ε	0.2	Ecart-type du processus de revenu salarial
γ	0.217	Ratio dépense publique sur PIB
φ	0.082	Ratio de transferts sur PIB
λ	0.1	Probabilité du choix de dette
β	0.9647	Taux d'escompte

2.4 Résultats

Dans une économie où les marchés sont incomplets, où les agents subissent des chocs idiosyncrasiques sur le marché du travail et sont soumis à des contraintes de crédit, nous trouvons que le niveau de dette optimal fixé par la puissance publique sous la condition de cohérence temporelle est positif et élevé. L'étalonnage retenu délivre un ratio dette publique sur PIB optimal de 210%. Ce niveau est supérieur au niveau stationnaire et peut être mis en contraste avec les résultats de Krusell, Martin et Rios-Rull (2006) où un niveau de dette publique positif ne peut être clairement identifié. Ce résultat s'explique notamment par l'importance des anticipations formées par les agents. Dans un cadre sans engagement et où la puissance publique est cohérente dans le temps, les agents sont rationnels et savent que le niveau de dette publique est susceptible de changer au cours de périodes. De plus les agents sont capables de prédire correctement les éventuels choix futurs. Enfin les agents savent que leurs choix individuels peuvent modifier la dynamique de l'économie et les choix de dette futurs. Dans ce cadre, le niveau optimal de dette publique est atteint lorsque le processus de choix délivre au cours du temps le même niveau de dette.

Dans l'économie de référence, selon leurs situations individuelles, les agents opèrent des choix éventuellement différents les uns des autres. L'augmentation du niveau de dette publique a un effet immédiat de réduction de la pression fiscale à travers la baisse du taux de taxe. Cette baisse permet d'augmenter la consommation et donc le bien-être dans la période courante. Toutefois, ceci signifie également que la pression fiscale jusqu'au prochain choix de dette sera plus forte à partir de la période suivante. De la même façon, les agents doivent arbitrer les effets provenant de la variation des prix dans le temps. L'augmentation courante du taux d'intérêt faisant suite à la hausse de la dette publique permet d'aider les agents à réduire le coût de l'épargne de précaution et donc à améliorer leur bien-être. Toutefois, l'augmentation courante du taux d'intérêt provoque également une éviction du capital privé qui a un effet néfaste sur le bien-être. Ces effets sont contrebalancés dans le temps par la baisse future du taux d'intérêt et ce jusqu'à ce qu'un nouveau choix de dette puisse être opéré. L'impact des prix dépend également de la situation individuelle de l'agent. Les agents détenant un niveau d'actif élevé sont avantagés par une hausse du taux d'intérêt. Ainsi, le choix courant d'un agent résume l'ensemble des arbitrages temporels évoqués. La puissance publique à travers le critère utilitariste agrège les choix individuels et fixe le nouveau niveau de dette. Le niveau optimal trouvé correspond à la situation où aucun arbitrage temporel n'est possible en moyenne et où il est avantageux en moyenne de conserver la situation présente.

Conclusion

Ce chapitre se propose de déterminer le niveau de dette publique optimal dans une économie où la puissance publique est cohérente dans le temps et ne dispose pas de technologie permettant de garantir un engagement quant à ce choix de politique économique. Nous trouvons que le niveau optimal de dette est positif et élevé en présence de marchés incomplets, d'incertitude et de contraintes de crédit. Etant parfaitement rationnels, les agents anticipent les choix futurs et mesurent les conséquences de leurs décisions sur la dynamique de l'économie. Le niveau optimal de dette publique trouvé est le résultat de leurs arbitrages temporels sur les effets bénéfiques et néfastes de cette politique publique sur la pression fiscale et le niveau des prix.

Conclusion

Ce travail de thèse propose trois contributions originales pour évaluer le niveau de dette optimal dans une économie à marchés incomplets. Il avance également des éléments pour comprendre la façon dont ce niveau est fixé et pour déterminer l'impact de la dette publique sur les décisions individuelles. Le premier chapitre établit un état de la littérature sur la dette optimale. Le second chapitre propose une analyse de la dette publique optimale dans une économie où il existe des fluctuations macroéconomiques. Dans ce cadre, le principal résultat montre que le niveau de dette optimal a tendance à augmenter en présence de risque agrégé et qu'il existe des gains et des pertes le long du cycle économique. Le troisième chapitre considère la dette publique dans une économie où les individus peuvent être entrepreneurs ou salariés. Le résultat principal démontre que le niveau de dette optimal est plus faible dans un tel cadre qu'en l'absence d'entrepreneurs, puisqu'une partie du comportement d'accumulation des ménages salariés est remplacée par l'accumulation des entrepreneurs. Le dernier chapitre quitte le cadre des modèles stationnaires pour envisager l'analyse de la fixation de la dette publique dans le temps avec un modèle cohérent temporellement. Le niveau de dette optimal trouvé est positif et élevé. Ce niveau provient de l'arbitrage individuel des gains et pertes de l'augmentation du niveau de dette publique dans le temps et de l'agrégation de ces décisions par la puissance publique.

Public debt optimality in an economy with incomplete markets

Introduction

It is a fact that since the second half of the 1970's, there has been an unprecedented increase in the level of public debt both in absolute and relative figures. Focusing on the OECD countries alone, net government debt as a percentage of GNP rose from 20.7% in 1975 to 33% in 1985 and has continued increasing to 42.3% in 1995 and 43.6% in 2005 according to the OECD Economic Outlook (1985 and 2008).

With the notable exception of the traditional Keynesian school, public debt has often been regarded with a certain uneasiness in the history of economic thought. For instance, most classical authors agreed with the view that the government budget has to be managed in the same manner as a household budget and that public debt lead to wasteful expenditures. After the Keynesian revolution, in the decades following World War II, a positive level of public debt was regarded as a necessary instrument of economic policy. But it was only for a short period of time that such policies were broadly that well considered.

Even though David Ricardo's thoughts on the matter are rather close to those of other classical authors, he is nonetheless associated with Barro (1974), for the Ricardian Equivalence proposition, which says that under certain circumstances, collecting taxes or issuing public debt to finance public spending might be equivalent, leaving very little place for any public debt policy. Thereafter, the standard deterministic representative agent growth model has prevailed. In such a model and under lump-sum taxes, there is no role for public debt. If taxes are distorting, public debt can be seen as a way to smooth tax distortions over time. As to how much public debt is good, it is either undetermined or depend on a set of initial conditions (Barro (1979), Chamley (1985) and Chamley (1986)).

However, in the recent decades, many research agendas in macroeconomics have tried to provide better microfoundation to previous models and questions. Fiscal policy and public debt policy analysis have taken part in this process. As a result of these theoretical and methodological advances, the literature has tried to better apprehend and take into account micro level behaviors and phenomena such as financial intermediation, market completeness for insurance issues or idiosyncratic risk.

This doctoral dissertation can be included in this agenda. More precisely, the models presented here have to be related to a strand of the literature pioneered by Bewley (1980 et 1983) and then Huggett (1993) and Aiyagari (1994). The frameworks used in this dissertation explicitly model individual behavior such as idiosyncratic labor market productivity, interaction with financial intermediation through credit constraints or individual insurance against risk because of incomplete markets.

This dissertation has for main focus public debt and how its level should be set so as to be optimal for the economy. It extends the existing literature on public debt optimality in incomplete markets economies with three original contributions. With the seminal paper of Aiyagari and McGrattan (1998) as a building block, this dissertation explores the impact of aggregate fluctuations, entrepreneurial risk and time consistency of public policies on the optimal level of public debt. The framework used here provide novel results and intuitions that have to be contrasted with traditional results in the public debt literature.

This introduction consists of three parts. The first part provides a short survey of public debt in the economic literature. As the literature on public debt is vast, the survey will focus on key results related to the subject of this dissertation. I begin with an historical perspective of public debt optimality and proceed step by step to the recent contributions of the literature. The second part makes clear the motivations behind this dissertation. I argue that the Bewley (1980 et 1983), Huggett (1993) and Aiyagari (1994) strand of the literature is relevant for the analysis of the optimal level of public debt. I also detail why a quantitative assessment of public debt is important. The last part provides an overview of the three chapters of the dissertation.

3 Public debt optimality: a short survey

3.1 Public debt and the classical economists

Classical economists[2] regarded public debt with great circumspection and often opposed the views of the mercantilists. The mercantilists argued that the payment of interests on public debt does not leave the country and could be regarded as one set of the population transferring wealth to another set of the population, or as they phrased it "*the right hand which pays the left*". Thus, according to them, public debt has little economic effect.

Adam Smith (1776) for instance opposed this argument. According to him, public debt at their time was mainly an issue after periods of war. Governments were reluctant to increase their tax policy because they feared the negative impact of higher taxes on the public opinion of the war. Because a modern statistical toolbox was unavailable to those governments, they were also unable to estimate the correct amount of taxes to finance war. As a result, they used public debt. But because of the accumulation of public debt and the necessary extension of the repayments to the peace era, the government budget was seldom balanced. Adam Smith argues that governments should not run budget deficits and that the accumulation of public debt is "*pernicious*" because of increased taxation, flight of capital and devaluation of the currency. He clearly states that public debt retards "*the natural progress of a nation*

[2]For a detailed survey on classical economists and public debt see Tsoulfidis (2007).

towards wealth and prosperity". Interestingly, he distinguishes between taxation that decreases to a certain extent private expenditure but has only a limited impact on future accumulation of capital and public debt that crowds out an identical amount of private investment and thus impacts the existing productive capacities.

Thus, for Adam Smith, the optimal level of debt should be as close as possible to zero and all government consumption should be financed by taxation except in the case of emergencies such as war. Adam Smith's view was shared by David Hume. De Haan (1987) quotes Hume in a particularly explicit manner: "*either the nation must destroy public credit or public credit will destroy the nation*". Hume argues that there should be no difference of conduct between the government and an individual when it comes to public debt. Hume thought that public debt led to wasteful expenditures and opted when possible for an optimal level of debt close to zero.

The views of David Ricardo on the subject are more difficult to apprehend mainly because of the so called Ricardian Equivalence principle named after him. In fact, Ricardo's view was in many cases very similar to the authors above. Tsoulfidis (2007) quotes Ricardo:

> "*[The system of borrowing] is a system which tends to make us less thrifty to blind us to our real situation. If the expense of a war be 40 millions per annum, and the share which a man would have to contribute towards that annual expense were 100l., he would endeavour, on being*

> *at once called upon for this portion, to save speedily the 100l. from his income. By the system of loans, he is called upon to pay only the interest of this 100l., or 5l. per annum, and considers that he does enough by saving this 5l. from his expenditure, and then deludes himself with the belief, that he is as rich as before.*"

It is clear to Ricardo that people do not perceive public debt and taxes as equivalent and thus take different economic decisions depending on the way public expenses are financed. His position would then be that, based on purely economic principles, either ways are equivalent but because of public perception, it should not be considered equivalent. Nevertheless, he persists in saying that public debt should be kept to a minimum because of the adverse effects of public borrowing on current productive capital accumulation. As long as government expenditures are not productive, the optimal level of debt should be close to zero.

This view is also shared by John Stuart Mill. Because the classical economists above generally followed Say's Law, they thought that general overproduction of good was not an issue. However, Malthus argues that overproduction could be a serious issue. In such a case, a positive level of public debt could be useful because by increasing current demand, it could help to solve the excess supply problem. Also, all classical economists above seem to agree on the fact that if public debt finances productive government spending, there might be a role for it.

However, it seems, as reported by Buchanan (1958) that the classical formulation of a public debt theory is best represented by the works of Adams (1887), Bastable (1895) and Leroy-Beaulieu (1906). Buchanan (1958) reports that theses authors and particularly Leroy-Beaulieu (1906) carefully point out past misconceptions such as the mercantilist notion of "*the right hand which pays the left*" or the english classical economists' view of non productive public spending. In the end, the conception of public debt in the early twentieth century can be summarized by this quotation of Leroy-Beaulieu (1906) translated by Buchanan himself:

> "*A loan will be useful or harmful to the society in general depending on whether the state preserves and usefully employs the proceeds, or wastes and destroys the capital which the rentiers have given up. In the past, the passions of sovereigns and the mistakes of governments have had for an effect the disbursing of the greater part of the proceeds of public loans for useless expenditures. This has led many to condemn public credit absolutely, as an instrument of evil. This conclusion is exaggerated. It is as much as to say that it would be desirable for a man to be without sense because he often does not use it properly.*"

3.2 Public debt and Keynesian principles

Most classical principles of public debt waned with the so called *Keynesian revolution*. Although it appears that Keynes himself was not inclined toward public deficits as an efficient tool for stabilization policies[3], after World War II fiscal policy was considered to be an effective instrument of economic policy. Even today, through the persistence of the IS-LM model,[4] we have an inheritance of the Keynesian era.

The IS-LM model can be seen as an aggregation of Keynesian and more classical ideas. IS-LM is a tool that applies demand side economics for stabilization policies and introduces a multiplier effect of an economic policy on the output. A general conclusion of the model (under a set of hypothesis) is that public spending can decrease unemployment in an efficient way if it is financed by public deficits. Although it is based on rigid hypothesis, the balanced budget theorem (Haavelmo(1945)) states that it is also possible to obtain an effect on unemployment by the means of taxation but generally with a lower multiplier effect. However, in the Keynesian view, a positive level of public debt was not particularly seen as a major problem for the economy. By the end of the seventies, the Keynesian theoretical framework had lost most of its grasp on the economy. Slow growth, inflation and the emergence of the new classical economics reduced the impact of Keynesian principles and with it the perception of public debt.

3.3 Public debt neutrality

The question of the neutrality of financing public expenses by the means of taxation or public debt is important for optimal public debt issues. Before Robert Barro

[3]See for instance Kregel (1985).
[4]See Colander(2004) for an essay on the persistence of the IS-LM model.

exposed the so called *Ricardian approach*, the standard approach[5] to budget deficits was the following. Substituting public deficit for taxation led to an increase in consumption and a decrease in *desired* private saving in terms of investment demand in a closed economy. In order to restore the balance between *desired* private saving and investment demand, an increase in the interest rate was needed. As a result, private investment was crowded out and the long-run stock of productive capital was smaller. Public debt was perceived as an intergenerational burden because it led to a smaller stock of capital for future generations.

According to Barro (1974), in an overlapping generations model, households fully perceive that the present value of future taxes equals the current value of the corresponding public debt. This only holds under the assumptions of intergenerational altruism and lump-sum taxation. Those conditions imply that agents act as if they were infinitely lived and yield that households do not perceive any increase in net wealth of a higher level of public debt. As a result, national saving is left unchanged and there is no effect on the interest rate of an increase in public debt in a closed economy. Public debt is neutral as long as there is no transaction cost for bond issue and tax collection. This argument, also called the *Ricardian equivalence principle*, rules out the concept of public debt optimality.

Barro (1979) considers the role of long run public debt with tax distortions taking the form of a quadratic tax collection cost function. The main result of this paper is

[5]See for instance Modigliani (1961) for further details on this approach.

that to minimize tax distortions over time, the tax to GDP ratio should be equal in all the periods considered. This result implies in general that the long-run level of public debt is indeterminate because it depends on initial conditions being mainly the initial level of public debt at the beginning of the policy horizon. Thus there is no place for public debt optimality in this model either. This result is further extended in Chamley (1985).

However, the *Ricardian equivalence principle* has been criticized many times. For the matter at hand, an extremely interesting point is made by Barsky, Mankiw and Zeldes (1986). These authors take a typical ricardian setting à la Barro and introduce future income risk and proportional taxes into the model. Those ingredients yield that a tax cut today financed by public debt and future higher taxes is enough to boost current consumption and rules out the neutrality result. The intuition is the following. A tax cut today provides certain wealth whereas future tax increase is contingent upon future income. Through the tax policy the government provides insurance to individuals that is not available in the private markets. Because the tax rate is proportional, the future tax increase is contingent on future income. In the next period the contribution to total taxes of a poor household would be lower than that of a rich household. Thus, the government reduces the variance of future income. In such a situation, an agent is more likely to use the tax cut to reduce precautionary saving and increase consumption because future tax burden pools future income risk. This is to be compared to the lump sum tax case where such insurance is not present. This paper is also noteworthy because its result relies on the absence of contingent claims markets to insure against future risk. The authors argue that the complete market hypothesis is indeed fictional and a model of "*missing markets*" or namely an incomplete market hypothesis is key to produce the results.

3.4 Public debt and incomplete markets

The role of government debt and its welfare effects in this branch of the literature contrast with the results above. The market incompleteness presented here relates to the works of Bewley (1980, 1983) on the optimal quantity of money and Huggett (1993) and Aiyagari (1994) on interest rates and aggregate saving. Credit constraints and heterogeneous agents are introduced to outline imperfect financial intermediation and uncertainty.

3.4.1 The Woodford (1990) model

The Woodford (1990) model is the first to consider imperfect financial intermediation to explain that a high level of public debt might be beneficial to the economy in terms of welfare. The results in this model are contrasted to what Woodford (1990) calls the "*neoclassical model*". The "*neoclassical model*" refers to the general equilibrium version of the traditional "*life cycle model*". In such a model, a higher level of public debt will reduce national saving and capital accumulation and in turn reduce welfare in the long run. This view also rules out Ricardian principles because of their apparent contradictions with notable economic observations. Woodford (1990) argues that although some predictions of the "*neoclassical model*" are correct, its framework might not be adequate for the analysis of public policies.

According to Woodford (1990), the Ricardian equivalence fails because of imperfect financial intermediation, namely the existence of borrowing constraints. In such a situation, the government debt can ease the borrowing constraint of agents by providing liquid assets to the economy in exchange for future taxes. Thus government debt in not neutral anymore. Woodford (1990) argues that the liquidity constrained economy is more consistent with empirical evidence such as the persistent low real return of U.S. public debt relative to other assets. He also emphasizes that the liquidity constrained model preserves the non neutrality of public debt even with altruistic bequests across generations, and thus does not fall to the Barro critique.

The Woodford (1990) model considers heterogeneously endowed households enduring borrowing constraints to show that a high level of debt is efficient because of the existence of a borrowing constraint. The economy is made of two types of households. Each household receives an endowment at each period. One type of households receive a low endowment in all even (bad) periods and a high endowment in all odd (good) periods. The other type of households has an opposite endowment scheme. When households are in a good period, they are not borrowing constrained and can save by holding government bonds or by accumulating capital, both yielding the same interest. However, if households are in a good period today, they were in a bad period yesterday and thus they were borrowing constrained and did not save in the previous period. When households are in a bad period, they are borrowing constrained and do not save. However, they receive the return on their previous period savings.

The author compares alternative stationary equilibria given a level of outstanding public debt per capita. He shows that by appropriate variable identification, the model can me made similar to an overlapping generations model such as Diamond (1965) and has similar properties concerning the effects on the real interest rate of a change in the size of government debt: if consumption is sufficiently substitutable over time, a higher stationary debt will be associated with a higher level of real interest rate and with a lower stationary capital stock. Thus, a higher level of debt crowds out private capital.

However, the welfare consequences are not identical in the two models. Here, an efficient allocation requires that the liquidity constraint does not bind. This is only possible if the interest rate is close to the time preference rate. Consequently, the main result of the paper is that efficiency requires that the real interest rate should be kept high enough, which means that the outstanding public debt has to be maintained at a high enough level[6]. This level of interest rate is higher than in the neoclassical model of Diamond (1965). It is possible that the equilibrium

[6] In the case of a large enough intertemporal elasticity of substitution.

remains inefficient if an increase in the level of debt affects the interest rate, savings or investment. There is thus a *liquidity effect* of public debt: it might be optimal to increase the size of the public debt to the point where liquidity constraints cease to bind.

Woodford (1990) also argues that by appropriately modifying his framework, it is possible that public debt "*crowds in*" private capital. To obtain this result the author assumes that households have access to the production technology only at certain periods instead of every period. He interprets this as the fact that interesting investing opportunities happens only from time to time. Woodford (1990) shows that if all households are borrowing constrained in the periods they have access to an investment opportunity, the steady-state capital stock varies in the same direction as the real interest rate for a given increase in public debt.

The model in Woodford (1990) is an important step from the model in Barsky, Mankiw and Zeldes (1986) to understand how the level of public debt should be set. Using imperfect financial intermediation, Woodford (1990) shows why Ricardian equivalence does not apply and why public debt should be set at a level high enough to reduce borrowing constraints. However, this model provides no real quantification of the optimal level of public debt.

3.4.2 The Aiyagari and McGrattan (1998) model

Aiyagari and McGrattan (1998) is an extension of Aiyagari (1994) with optimal public debt as its main focus. It is the first paper to address quantitatively the question of the optimality of public debt in a calibrated model. It is a Bewley-Huggett-Aiyagari type dynamic stochastic general equilibrium model. Insurance markets are incomplete, households face borrowing constraints and idiosyncratic shocks on their labor income. When insurance markets are complete, the model collapses to the representative agent growth model except for the idiosyncratic labor productivity process.

The economy is populated by a continuum of infinitely-lived *ex-ante* identical and *ex-post* heterogeneous agents of measure unity. The production sector is populated by a continuum of firms which have a neoclassical production technology and behave competitively in product and factor markets. Households supply labor elastically to the production sector and face uninsurable labor income risk. They also accumulate a safe asset that can be either private capital or public debt claims. Both yield the same return. Households also face a strict borrowing constraint. The government issues public debt and levies taxes to finance public expenses. The revenues of labor and returns on capital assets are taxed proportionally at an identical rate. The main calibration objective here is to reproduce the U.S. capital-output ratio. Aiyagari and McGrattan (1998) do this by adjusting the discount rate.

The results of this paper are depicted in Figure 1. The interest rate increases when public debt increases and there is a crowding out effect of debt. Hours worked are decreasing and the tax rate varies non monotonously. The optimum quantity of public debt found by Aiyagari and McGrattan (1998) is about 66% of GDP and is close to the average postwar debt to GDP ratio computed by the authors. This positive level is explained by two opposing effects. On the one hand, because of the idiosyncratic risk they face, agents engage in precautionary saving. This is the only way agents can reduce the risk they face as insurance markets are incomplete and borrowing constraints might bind. However, precautionary saving has a welfare cost. In order to accumulate more precautionary saving, agents have to postpone current consumption, what in turn reduces their welfare level. In this model, public debt by increasing the interest rate reduces the cost of postponing consumption i.e. reduces the cost of precautionary saving.

On the other hand, a higher level of public debt crowds out private capital and through higher taxation distorts labor supply and saving decisions. This directly reduces household welfare. The optimal level of public debt found by Aiyagari and McGrattan (1998) is the level such that each of those two opposing effects exactly balances the other. Aiyagari and McGrattan (1998) also find that the welfare profile in their economy is flat. If a policy was undertaken so as to reach a debt to GDP ratio of zero percent instead of the benchmark level of 66%, the welfare loss would be about 0.08% of consumption.

Aiyagari and McGrattan (1998) also isolate the effect of distortionary taxation

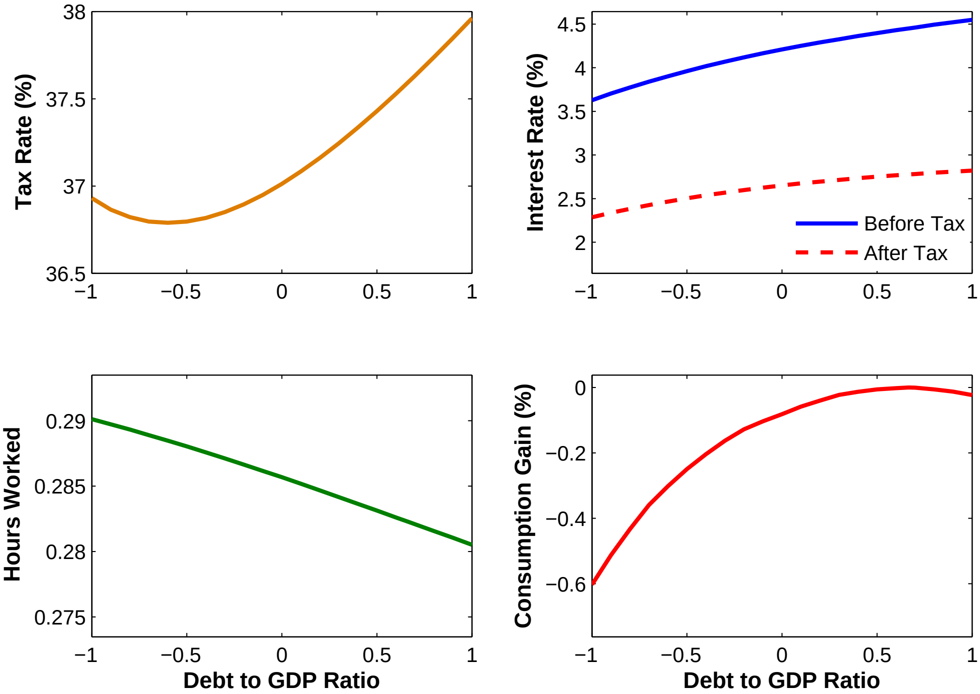

Figure 1: Aiyagari and McGrattan (1998) results

by comparing their benchmark results to those of a model with lump-sum taxation. With a lump-sum tax, the only opposing effects are the crowding out of private capital and the reduction of the cost of precautionary saving. When the discount factor is properly adjusted to deliver comparable equilibrium interest rates in the two models, the optimal quantity of debt is 1.4 times GDP. We can deduce from this that tax distortions plays an important role. However, the welfare profile remains flat in this alternative model. The welfare gain to be at the optimal level rather than the U.S. level is around 0.22% of consumption.

The Aiyagari and McGrattan (1998) paper is the starting point of this PhD essay. This essay criticizes some aspects of their paper and extends some others. For instance in Aiyagari and McGrattan (1998), aggregate risk is ruled out by assumption. Also, the optimal level of public debt found relies on the accumulating behavior of working households where empirical studies reveal that a large part of national saving is done by entrepreneurs. Moreover, some calibration aspects of Aiyagari and McGrattan (1998) can be improved. This paper only targets the U.S. capital-output ratio. As a consequence, as reported by Table 2, the reproduction of wealth statistics is poor. Finally it is also possible to extend this model to consider alternative hypothesis than the steady state analysis provided by Aiyagari and McGrattan (1998).

Table 2: Wealth distribution in Aiyagari and McGrattan (1998) model

Model	Capital-output ratio	Wealth Gini	Percentage wealth held by top			
			1%	5%	10%	20%
Aiyagari and McGrattan (1998)	2.5	0.42	4	15	26	44
U.S. Data	2.5	0.78	29.5	53.5	66.1	79.5

Motivations

One of the earliest motivation behind this doctoral essay, was to focus on public debt policies and public debt optimality, leaving the representative agent, complete market framework behind. The representative agent, complete market framework gives essential insights on many economic issues but it also falls short on some others[7]. Among other shortcomings, in the case of public policies, a representative agent framework does not allow thorough distributional analysis.

The complete market hypothesis is also problematic. Under this assumption, an agent can find a contract today to insure against any risk he might encounter tomorrow. Apart from being unrealistic, this assumption rules out a very factual behavior of economic agents: precautionary saving. In the absence of such insurance contracts, namely when markets are incomplete, the economic agents have to use every means they have to transfer more wealth into the future to insure against future risk. Most of the time, the only asset readily available to do so is private saving. Thus, when economic agents abstract from market completeness, there is a steady increase in the amount they save depending on the magnitude of the risk they are facing. As this saving serves the purpose of insurance against unknown future risk, it is called precautionary saving.

However, even when markets are incomplete, economic agents might use another way than precautionary saving to insure against future risks: they might infinitely borrow. Despite the fact that markets are incomplete, if an agent can just borrow any amount of money he needs at any time, he can face any future risk he might encounter by borrowing more. In the real world, the intermediation sector has stringent rules on how much it can lend to an individual. Other means of borrowing are also limited or subject to collateral. These rules and limitations need to be made

[7]As it is not the subject here, a critique of the representative agent model can be found in Kirman (1992).

explicit. This is done by introducing borrowing constraints in the model that sets the amount an agent can borrow.

Empirical evidence suggests that a large proportion, something between 10 and 20 percent in the U.S., of households are borrowing constrained (Hall and Mishkin (1982), Hayashi (1985), Mariger (1986), Hubbard and Judd (1986), Jappelli (1990) or Cox and Jappelli (1993)). Jappelli (1990) for instance reports, using data from the 1983 Survey of Consumer Finances, that 19% of families are rationed in the credit market. These households account for 12.7% of total income, 7% of wealth and are on average younger than the rest of the population. The existence of borrowing constraints is important to understand household decisions. If an adverse shock makes an agent borrowing constrained, not only will his current consumption be impacted but also he will be unable to transfer wealth into the future. Being borrowing constrained is a worst case scenario for an agent. Thus, because this possibility is anticipated, he will take every possible measure to avoid being constrained. If no other insurance technology is available, borrowing constraints give a strong motive for precautionary saving.

Another necessary ingredients is the incomplete market hypothesis. Carroll (1997) reports that 43% of the consumers who participated in the 1983 Survey of Consumer Finances said that being prepared for emergencies was the most important reason for saving. In contrast only 15% said that preparing for retirement was the most important saving motive. This is one of many evidence that actual agents do not have access to contingent claim markets to insure against future risk. If future income is uncertain then agents will seek a way to insure this risk. Not only will they not find a complete set of insurance contracts to fulfill this urge, but in the best case with the available markets and private saving they will only be able to partially insure against future risk. Part of the risk they face will not be insured and thus anything that might better insure or reduce the cost of current insurance is welcome.

For instance, Barsky, Mankiw and Zeldes (1986) show in a typical ricardian setting à la Barro modified to allow future income risk and proportional taxes that a tax cut today financed by public debt and future higher taxes is enough to boost current consumption. A tax cut today provides certain wealth whereas future tax increase is contingent upon future income. Because the tax rate is proportional, in the next period the contribution to total taxes of a poor household would be lower than that of a rich household. Thus this government policy reduces the variance of future income. In such a situation an agent is more likely to use the tax cut to reduce precautionary saving and increase consumption because future tax burden pools future income risk. This is to be compared to the lump sum tax case where such insurance is not present. As a matter of fact, through the tax policy the government provides insurance to individual that is not available in the private markets. If markets were complete, such adjustments would not be necessary.

The precautionary saving motive is now well described and relatively not disputed in the literature. The precautionary motive can be dated back to at least Keynes (1936) and has been used to answer shortcomings of the life cycle model on consumption and saving. In the standard life cycle model, current consumption is only correlated to unanticipated future income variations. However, empirical evidence (see for instance Flavin (1981) or Hall and Mishkin (1982)) suggests that consumption reacts to anticipated income variations. The precautionary saving motive is key to reconcile the empirical evidence and the model. As it has been shown by Zeldes (1989), Kimball (1990), Carroll (1997) and numerous other papers, precautionary saving is a means of insurance against future adverse shocks and can be considered central in the behavior of a typical household.

Precautionary saving is also important to understand the way public debt impacts individual decisions. Although precautionary saving serves an insurance motive and thus is used to smooth consumption over time, it has a cost that is precisely the amount of current consumption one has to give up. This cost can be reduced if the interest rate can be made higher because the return on saving would be higher. As a result a lower amount needs to be saved, or equivalently, more current consumption is possible for the same quality of insurance against adverse future risk and consumption smoothing. This is specially true for households with a large capital component. For households depending on labor income, a higher interest rate might be not interesting because of the subsequent wage decrease. Changing the level of public debt in the economy has an impact on the interest rate and affects the cost of precautionary saving and therefore welfare.

The first author to depart from the representative agent complete market framework in order to introduce heterogeneity and market incompleteness is Bewley (1980 and 1983). He builds a heterogeneous agents, incomplete markets economy with borrowing constraints to assess the optimal quantity of money. In this model, households engage in precautionary saving and *ex ante* identical agents become *ex post* heterogeneous because of different trajectories on the labor market and subsequent different stories of asset holding. The first author to introduce uninsurable risk and credit constraints in the analysis of public debt is Woodford (1990). Using a simple model he shows that a positive amount of public debt can be efficient because higher levels of public debt keep interest rates closer to time preference rates. However there is no quantitative assessment of the optimal level of public debt.

The second motivation for this doctoral essay was to provide a quantitative assessment of the optimal level of public debt, and, although it was clear from the start that it would be difficult to be exhaustive, to make explicit key elements that have an impact on the optimal level of debt. The seminal work of Aiyagari and McGrattan (1998) was the main building block supporting this motivation. Aiyagari and McGrattan (1998) use a heterogeneous agent, incomplete market setup where agents are subject to borrowing constraints to quantify the optimal level of public debt. Their framework reproduces the U.S. capital-output ratio and yields an annual optimal level of public debt of 66%. This level coincides with the average postwar level of U.S. public debt.

The main intuition behind this result is the following. Public debt has opposing effects on the welfare of economic agents. On the one hand, higher levels of debt crowd out private capital and thus decrease welfare. On the other hand, higher levels of public debt raise interest rates and help agents to reduce the cost of precautionary saving. This last effect helps the consumption smoothing behavior of economic agents and is welfare increasing. According to Aiyagari and McGrattan (1998), the latter effect exactly balances the former for a ratio of debt of 66% which can be then said to be optimal.

Aiyagari and McGrattan (1998) is the first paper to give a quantitative assessment of the optimal level of public debt. It is also the first paper to give an essential insight on the way public debt influences individual decision and welfare in presence of uncertain income risk, market incompleteness and credit constraints. Among the mechanisms described by Aiyagari and McGrattan (1998), precautionary saving plays a key role in explaining the impact of public debt on the economy. Credit constraints and incomplete markets rule out the ricardian equivalence principle. Thus, the results of the paper differ in many points from the traditional results in the public debt literature.

But at the same time, Aiyagari and McGrattan (1998) ignore or rule out other important aspects of the economic environment. Floden (2001) for instance, extends the Aiyagari and McGrattan (1998) framework to study the optimality of public debt and transfers in combination. He finds that the optimality of public debt is strongly correlated to the optimal level of transfers. Precisely, when transfers are at the optimal level, Floden (2001) reports that the optimal level of debt need not be as high as the level found in Aiyagari and McGrattan (1998).

This doctoral essay tries to overcome other shortcomings of the literature concerning public debt in incomplete markets economies. Barsky, Mankiw and Zeldes (1986) indicate that aggregate uncertainty is the main reason driving uncertainty of future individual income and since then a large literature has developed around aggregate uncertainty and its correlation with individual uncertainty issues. This aspect is ruled out by assumption in Aiyagari and McGrattan (1998).

Being a calibrated model with quantitative focus, Aiyagari and McGrattan (1998) fall also short on reproducing key elements of the wealth distribution of the target economy. This is not neutral when assessing public debt optimality. As it is discussed in Floden (2001) or Ball and Mankiw (1995), public debt mostly benefits wealthy households because they gain from higher return on capital when poorer people suffer from higher taxes and lower wages.

The income process used in Aiyagari and McGrattan (1998) can also be criticized. It is not persistent enough and focuses on the income of agents depending on the labor market. Empirical studies reveal that a large part of national saving is done by entrepreneurs bearing different income risks than that of workers so that it is important to consider this category of agents for public debt policies.

Finally, the optimal level of debt characterized by Aiyagari and McGrattan (1998) is a steady state level. It is also interesting to consider alternative hypothesis on this matter and ask how the government would set optimal public debt if the choice of public debt could be endogenous and repeated.

Overview

This doctoral essay extends the literature on public debt in incomplete markets economies following the work of Aiyagari and McGrattan (1998). It is made of three chapters that constitute original contributions to the literature. The first chapter reconsiders the optimal level of public debt in an economy with aggregate fluctuations. The cost of business cycles is a subject that has received great interest, particularly since Lucas (1987). Following a suggestion in Imrohoroglu (1989), the paper looks at the interaction between public debt policies and the business cycle and builds a framework exhibiting aggregate fluctuations to quantify the optimal level of public debt. It is argued that as aggregate fluctuations have an effect on the saving behavior of economic agents, it should be an important element to be taken into consideration for public debt policies.

Also, some steps are taken to reproduce key elements of a target economy such as the capital-output ratio, the wealth distribution or the wealth Gini. A production economy encompassing capital market imperfections where a large number of ex-ante identical infinitely-lived agents face idiosyncratic income shocks and aggregate productivity shocks constitute the benchmark economy. Households' saving behavior is influenced by precautionary saving motives and borrowing constraints. Private capital and government bonds, both yielding the same interest, can be claimed to insure against future risk. Government levies proportional taxes on households and issues debt in order to finance its consumption. The main result is that the optimal level of debt is positive and higher than in a model without aggregate risk. Also, gains and losses of higher levels of public debt are different along the business cycle or across the wealth distribution.

The second chapter introduces an economy where both entrepreneurial households and standard working households are subject to long run variations of the level of public debt. I emphasize that Aiyagari and McGrattan (1998) may have overestimated the optimal level of public debt because their result relies only on the strong saving behavior of workers. Empirical evidence shows that although entrepreneurs constitute a small fraction of the economy, they have a critical impact on the national saving rate. Their portfolio is poorly diversified and tends to be biased toward their private business assets: Moskowitz and Vissing-Jorgensen (2002) find that 45% of entrepreneurial wealth is invested in their business assets. However, recent evidence also suggests that entrepreneurs engage in precautionary saving behavior as a means of insurance (Gentry and Hubbard (2004) and Covas (2006)).

This chapter models a stylized economy with both entrepreneurial and non-entrepreneurial households subject to uninsurable risk. As it is traditional with Bewley-Huggett-Aiyagari class of models, I consider borrowing constraints and market incompleteness. The economy has both a corporate production sector and an entrepreneurial production sector. The government levies proportional taxes and issues public debt in order to finance public consumption. The benchmark calibration parameters are chosen to make comparison easier with the standard results of Aiyagari and McGrattan (1998).

This setup yields a negative steady state level of optimal public debt and matches the U.S. wealth distribution and wealth Gini. I show that this result comes from the fact that part of the accumulation behavior of working households is transferred to the entrepreneurial households. This is empirically relevant. When public debt increases, because workers accumulate less, they are more concerned by the adverse effects of crowding out and rising taxes than the beneficial effect of the reduced cost on precautionary saving. Because they are subject to high levels of risk, entrepreneurs on the other hand prefer higher level of public debt. They engage in precautionary saving and higher interest rates help to reduce the cost of postponing consumption. In the end, the optimal level of public debt in the economy is explained by the two opposing effects of debt on workers and entrepreneurs. Because workers are in larger proportions in the economy, the optimal level of public debt is negative at the steady state.

The third and last chapter moves away from the steady state comparison approach of the two previous chapters and previous literature. We consider the importance of time consistent policies for the optimal level of debt in an incomplete market framework. A time-consistent policy describes successive forward-looking governments playing a game against each other in time. This paper argues that the assumptions of commitment, market completeness and uncertainty have a strong impact on the level of optimal public debt and on the way it is chosen over time. The empirically relevant feature is that almost all governments end up with large positive amounts of public debt. On the one hand and to the best of our knowledge, models without commitment remain in the simplifying assumptions of complete markets and representative households and are unsuccessful in explaining the observed positive amount of public debt. On the other hand, models with incomplete markets fail to capture the important behavior of choosing debt over time and its implication on the optimal level of public debt.

Thus, this paper builds the simplest possible setting to relax these assumptions in order to assess the optimal level of public debt. We find the time-consistent optimal level of debt to be large and positive. The intuition behind this result is the following. In a time-consistent equilibrium, agents are fully rational and can predict the outcome of future debt choices. Moreover, agents make their own choices knowing the effects they will have on the evolution of the economy. Increasing the level of debt today has the immediate effect of bringing the tax rate down. As it will boost current consumption, this is welfare increasing. But agents know that tomorrow and the consequent periods, they will have to pay higher taxes until a new choice can be made. Similarly a higher level of debt has an effect on prices. The interest rate immediately increases but falls down in the future. Higher interest rates can help agents to reduce the cost of precautionary saving and increase welfare. But as higher levels of debt crowd out private capital, agents have also to endure welfare losses. Agents will choose higher levels of debt until the immediate gains are equal to future losses. Eventually, the benevolent planner will set a level of debt that maximizes total welfare in the economy.

Chapter 1

Public debt and aggregate risk[1]

[1]This chapter is a joint work with Audrey Desbonnet

Introduction

The introduction of uninsurable risk was a major step in the recent literature on public debt. Woodford (1990) is the first author to depart from the complete market framework to consider credit constraints or as he phrases it "imperfect financial intermediation". Woodford (1990) builds a simple economy embedding credit constraints and finds that public debt can be efficient because it keeps interest rates higher and closer to time preference rates. Aiyagari and McGrattan (1998) quantitatively address the question of the optimal level of public debt in a heterogeneous agent, incomplete market model. Calibrating on the U.S. economy they find the annual positive debt over GDP ratio of 2/3 to be optimal. Finally, Floden (2001) uses a similar framework to look at public debt/transfers optimal combinations. This author underlines the strong uncertainty and inequality implications behind public debt policies.

Very much in the same way, the introduction of uninsurable risk in the literature on the cost of business cycles has generated rich implications. In an effort to reconsider the small welfare effect of business cycles found by Lucas (1987), both Krusell and Smith (2002) and Storesletten, Telmer and Yaron (2001) draw a link between the aggregate risk and the cross sectional distribution. In other words, the aggregate productivity shock is correlated to the individual specific shocks to produce strong distributional effects of aggregate fluctuations. In such a framework, these authors find a greater cost of business cycles than Lucas (1987). Even more interestingly, Imrohoroglu (1989) briefly suggests that economic policies could be used to help individuals reduce the cost of business cycles. In this chapter, we explore this latter suggestion and its implications for the optimal level of public debt.

This chapter's main objective is to introduce a framework exhibiting aggregate fluctuations à la Den Haan (1996) or Krusell and Smith (1998) to quantify the long-run optimal level of public debt. As aggregate fluctuations impact the saving behavior of agents, they are an important aspect to take into account for public debt policies that has been ignored so far. Unlike previous literature, this chapter also takes some steps to reproduce the wealth distribution of the targeted U.S. economy as it has been documented that the need for public debt is very different across the population. Thus our framework can be used to decompose the effects of public debt along the cycle and also across the population.

Following Bewley-Huggett-Aiyagari type models, we build a production economy with capital market imperfections where a large number of ex-ante identical infinitely-lived agents face idiosyncratic income shocks and aggregate productivity shocks. Households' saving behavior is influenced by precautionary saving motives and borrowing constraints. Private capital and government bonds, both yielding the same interest, can be claimed to insure against future risk. Government levies proportional taxes on households and issues debt in order to finance its consumption.

In this setup, our benchmark calibration yields a long-run optimal level of public debt of 5% of GDP on an annual basis. This is to be compared to a model without aggregate fluctuations. We consider several methods of removing aggregate fluctuations to obtain a comparable long-run idiosyncratic risk model. The level of public debt obtained in those idiosyncratic models are generally lower although with one method the optimal level of debt is almost similar to the aggregate risk model. Thus we argue that the effect of aggregate risk on the optimal level of public debt is moderate. In the benchmark economy, we find that the gains of being at the optimal level of debt instead of the benchmark debt to GDP ratio of 66% amounts to 0.257% of consumption. Also, the consumption gains of being at the optimal level of debt are higher in recessions than in expansions and higher for the poorest percentile of the population.

The intuition behind our results is the following. Credit constraints and uncertainty lead agents to engage in precautionary saving. The result of precautionary saving is a higher level of capital that in turn lowers the interest rate away from the time preference rate. Aggregate fluctuations, as it is correlated with the labor market process, exacerbate the level of risk faced by agents. An *employment fluctuation effect* increases both the risk of losing one's job and the time one could spend in unemployment in recessions. This strengthens households' precautionary saving motive. A *price fluctuation effect* changes the level of prices between recessions and booms making it more costly to accumulate precautionary saving in recessions. As a result of these effects, the capital stock rises and the interest rates move further away from the time preference rate.

A higher level of public debt has two opposing effects here. A *crowding out effect* crowds out private capital and reduces welfare. A *cost of precautionary saving effect* increases the interest rate, reduces the cost of precautionary saving and enhances welfare. Because the *employment fluctuation effect* and the *price fluctuation effect* change the precautionary saving motive and cost, the *cost of precautionary saving effect* is stronger in an aggregate fluctuations setting than in an idiosyncratic risk setting. Thus the optimal level of public debt is higher in a setting with aggregate fluctuations. Finally, we emphasize that the optimal level of debt can be significantly higher if we modify the parameters governing the *employment fluctuation effect* so as to strengthen this effects.

The rest of the chapter is organized as follows. Next section describes the benchmark economy. Section 2 details the results. In section 2.5 we characterize the optimal level of public debt for the European labor market. The last section concludes.

1 The Benchmark Model

Our benchmark economy is a Bewley-Huggett-Aiyagari type dynamic stochastic general equilibrium model augmented to allow aggregate fluctuations à la Den Haan (1996) or Krusell and Smith (1998) and public debt. Insurance markets are incomplete. Agents face idiosyncratic and aggregate risks and are borrowing constrained. These three assumptions lead agents into precautionary saving (Aiyagari (1994)).

Our benchmark model can be related to the model in Aiyagari and McGrattan (1998) but presents three deviations of importance apart from its aggregate fluctuations feature. First, the productivity process here is simpler as agents can only be employed or unemployed. Second, leisure is not valued. Those two deviations greatly simplify the model with aggregate fluctuations. Finally, there is no exogenous growth in our benchmark economy. Aiyagari and McGrattan (1998) specify an exogenous annual growth rate of 1.85%. This assumption exogenously reduces the cost of public debt for individuals since public debt interest repayments are diminished by this exogenous growth factor. It is noteworthy that making this assumption lead to a higher optimal level of public debt.

1.1 Firms

We assume that there is a continuum of firms which have a neoclassical production technology and behave competitively in product and factor markets. The output is given by:

$$Y_t = z_t F(K_t, N_t)$$

where K is aggregate capital and N aggregate labor used in production. The function F exhibits constant returns to scale with respect to K and N, has positive and strictly diminishing marginal products, and satisfies the Inada conditions. Capital depreciates at a constant rate δ.

The economy is subject to an exogenous aggregate shock noted z. There are two possible aggregate states: a good state where $z = z_g$ and a bad state where $z = z_b$. The aggregate shock follows a first-order Markov process with transition probability $\eta_{z|z'} = \Pr(z_{t+1} = z'/z_t = z)$. Thus $\eta_{z|z'}$ is the probability that the aggregate state tomorrow is z' given that it is z today. We note η the matrix that describes the transition from one aggregate state to another such that:

$$\eta = \begin{pmatrix} \eta_{gg} & \eta_{gb} \\ \eta_{bg} & \eta_{bb} \end{pmatrix}$$

Finally, our setting assumes that inputs market are competitive. The wage w and the interest rate r verify:

$$r_t + \delta = z_t F_K(K_t, N_t) \tag{1.1}$$

$$w_t = z_t F_N(K_t, N_t) \tag{1.2}$$

1.2 The government

The government issues public debt and levies taxes to finance public expenses. Both the revenue of capital and labor are taxed proportionally at an identical rate τ. The government's budget constraint verifies:

$$G_t + r_t B_t + TR_t = B_{t+1} - B_t + T_t$$

with

$$T_t = \tau(w_t N_t + r_t A_t)$$

G_t is the level of public expenses, B_t the level of public debt, T_t tax revenues and TR_t a lump sum transfer to households that amounts to zero at the equilibrium. Because of the aggregate fluctuations property of the model, we have to be cautious about how we close the model with respect to the government budget constraint. We proceed as follows. As the aggregate variables are not constant, even in the limit because of the aggregate fluctuations property, we specify a *pseudo steady state* by averaging aggregate variables over long periods of time. We first guess an interest rate in this *pseudo steady state* and derive a tax rate by assuming that public expenses and public debt are a constant fraction of the associated long run GDP. As this tax rate does not necessarily balance the government budget constraint along the cycle, we use the lump sum transfer TR to make sure the budget will be balanced. Finally, we update our guess on the interest rate in the *pseudo steady state* until we reach a fix point. When this final step is completed, the model is closed with respect to the government budget constraint and TR amounts to zero at the equilibrium[2].

A_t accounts for total average wealth in the economy. It is the sum of average

[2]More details can be found in Appendix 1.

physical capital K and public debt B such that:

$$A_t = K_t + B_t \tag{1.3}$$

1.3 Households

The economy is populated by a continuum of *ex ante* identical infinitely lived households of unit mass. Their preferences are summarized by the function V:

$$V = \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \left(\prod_{j=0}^{t} \beta_j u(c_t) \right) \right\} \tag{1.4}$$

where β is the discount factor. We assume that this discount factor is random. Thus the discount factor β can differ across agents and can vary over time. We specify that the latter follows a three-states first-order Markov process. This assumption on the discount factor helps to reproduce the wealth distribution as shown in Krusell and Smith (1998). The discount factor verifies:

$$\begin{cases} \beta_0 = 1 \\ \beta_{j \geq 1} \in]0;1[\end{cases}$$

c_t is the household level consumption. The utility function we use has a standard

CRRA specification and writes:

$$u(c) = \begin{cases} \frac{c^{1-\sigma}}{1-\sigma} \text{ if } \sigma \neq 1 \\ \\ \log(c) \text{ if } \sigma = 1 \end{cases}$$

Agents are subject to idiosyncratic unemployment shocks. Let s be the household's labor market status. A household can either be unemployed ($s = u$) or employed ($s = e$). Households are also subject to shocks at the aggregate level. Aggregate shocks exacerbate idiosyncratic unemployment risks. The unemployment rate and the unemployment duration are higher in recessions than in booms. Therefore, transitions on the labor market are correlated to the aggregate state. We note $\Pi_{zz'|ss'}$ the joint transition probability to a state (s', z') conditional on a state (s, z). The matrix that jointly describes the transition from a state (s, z) to a state (s', z') is the following:

$$\Pi = \begin{pmatrix} \Pi_{bbuu} & \Pi_{bbue} & \Pi_{bguu} & \Pi_{bgue} \\ \Pi_{bbeu} & \Pi_{bbee} & \Pi_{bgeu} & \Pi_{bgee} \\ \Pi_{gbuu} & \Pi_{gbue} & \Pi_{gguu} & \Pi_{ggue} \\ \Pi_{gbeu} & \Pi_{gbee} & \Pi_{ggeu} & \Pi_{ggee} \end{pmatrix}$$

where $\Pi_{ggee} = \Pr(z_{t+1} = z_g, s_{t+1} = e | z_t = z_g, s_t = e)$.

When agents are in an employed state, they receive the wage w. However when agents are unemployed their income corresponds to their home production that we note θ. Insurance markets are incomplete so that agents can only partially self-insure against idiosyncratic risk. Following Aiyagari and McGrattan (1998) no borrowing is allowed. The only way for households to self-insure against idiosyncratic risk is to accumulate physical capital and government bonds both yielding the same return r. Their overall holding in the later assets is noted a. Therefore a typical household solves the following problem:

$$\max_{c_t,\, a_{t+1}} \mathbb{E}_0\{\sum_{t=0}^{\infty}(\prod_{j=0}^{t}\beta_j)u(c_t)\},$$

subject to:

$$a_{t+1} + c_t = (1 + (1 - \tau)r_t)a_t + \chi(s_t)w_t + TR_t$$

$$c_t \geq 0$$

$$a_{t+1} \geq 0$$

with

$$\chi\left(s_t\right) = \begin{cases} \theta \text{ if } s_t = u, \\ (1-\tau) \text{ if } s_t = e \end{cases}$$

The existence of aggregate risk leads us to distinguish between individual state variables and aggregate state variables. The individual state variables are given by the vector (a, s, β). The aggregate state variables are summarized by the vector (z, Γ) where $\Gamma(a, s, \beta)$ is a distribution of agents over asset holdings, employment status and preferences. To determine the wage and the interest rate, households need to forecast the aggregate stock of physical capital. Therefore, they need to know the wealth distribution. That is why wage and interest rate depend on that wealth distribution. In the computational appendix, we explain how we avoid manipulating the wealth distribution by approximating it with some of its moments using the methodology developed in Den Haan (1996) and Krusell and Smith (1998). We detail the computational strategy we used to solve the model in appendix 1.

1.4 Equilibrium

The recursive equilibrium consists of a set of decision rules for consumption and asset holding $\{c(a, s, \beta; z, \Gamma), a'(a, s, \beta; z, \Gamma)\}$, aggregate capital $K(z, \Gamma)$, factor prices $\{r(z, \Gamma), w(z, \Gamma)\}$, tax rate τ and a law of motion for the distribution $\Gamma' = H(\Gamma, z, z')$ which satisfy these conditions:

(i) Given the aggregate states, $\{z, \Gamma\}$, prices $\{r(z, \Gamma), w(z, \Gamma)\}$ and the law of motion for the distribution $\Gamma' = H(\Gamma, z, z')$, the decision rules $\{c(a, s, \beta; z, \Gamma), a'(a, \epsilon, \beta; z, \Gamma)\}$ solve the following dynamic programming problem:

$$v(a,s,\beta;z,\Gamma) = \max_{c,\ a'}\{u(c) + \beta E\left[v(a',s',\beta';z',\Gamma')|(s,\beta;z,\Gamma)\right]\}$$

subject to:

$$c + a' = (1 + r(z,\Gamma)(1-\tau))a + w(z,\Gamma)\chi(s) + TR$$

$$c \geq 0$$

$$a' \geq 0$$

and

$$\Gamma' = H(\Gamma, z, z')$$

(ii) Market price arrangements are:

$$\begin{aligned} r(z,\Gamma) &= zF_K(K,N) - \delta \\ w(z,\Gamma) &= zF_N(K,N) \end{aligned}$$

(iii) Government budget constraint holds.

(iv) Capital market verifies:

$$K + B = \int a'(a,\epsilon,\beta;\Gamma,z)d\Gamma$$

(v) Consistency: agents' optimization problem is satisfied given the law of motion

H and the law of motion is consistent with individual behavior.

1.5 Calibration

For the sake of comparison, the model economy is calibrated to match certain observations in the U.S. data. We let one period in the model be one quarter in the data. To avoid confusion we have converted all values to their annual equivalents in the results section. To remain simple and allow comparisons, we closely follow Krusell and Smith (1998) when calibrating the characteristics of the labor market and the aggregate risk.

1.5.1 Technology

We choose the production function to be Cobb-Douglas:

$$Y_t = z_t F(K_t, N_t) = z_t K_t^{\alpha} N_t^{1-\alpha} \quad 0 < \alpha < 1$$

Technology parameters are standard. The capital share of output α is set to 0.36 and the capital depreciation rate δ is 0.025. As Krusell and Smith (1998) we assume that the value of the aggregate shock z is equal to 0.99 in recessions (z_b) and 1.01 in booms (z_g).

1.5.2 Preferences and discount factor

In the benchmark economy we assume a logarithmic utility function. We now detail the calibration steps to generate a realistic wealth distribution, the observed U.S. wealth Gini index and the capital-output ratio. Here, our calibration differs from Krusell and Smith (1998). In their economy agents can borrow whereas here, for the sake of comparison and simplicity, we follow Aiyagari and McGrattan (1998) and no borrowing is allowed.

To reproduce the shape of the U.S. wealth distribution we first assume that unemployed agents receive income too and fix the home production income θ to be 0.10[3]. This assumption produces a large group of poor agents. Next we use the preference heterogeneity setting discussed in Krusell and Smith (1998) to generate a long thick right tail[4]. We impose that the discount factor β takes on three values $\{\beta_l, \beta_m, \beta_h\}$ where $\beta_l < \beta_m < \beta_h$:

$$\begin{pmatrix} \beta_l \\ \beta_m \\ \beta_h \end{pmatrix} = \begin{pmatrix} 0.9750 \\ 0.9880 \\ 0.9985 \end{pmatrix}$$

Thus an agent with a discount factor β_m is more patient than an agent with a

[3]This corresponds to about 12% of the average wage at the equilibrium.

[4]There are several ways to reproduce a thick right tail. One would be to give rich agents higher propensity to save or higher returns on saving for instance by introducing entrepreneurs (e.g., Quadrini (2000)). For the sake of simplicity we explore here the preference heterogeneity setting introduced by Krusell and Smith (1998).

discount factor β_l. To calibrate the transition matrix, we impose that the invariant distribution for discount factors has 10% of the population at the lowest discount rate β_l, 70% at the medium discount factor β_m and 20% at the highest discount factor β_h. As Krussell and Smith (1998) we assume that there is no immediate transition between extreme values of the discount factors. Finally, we set the average duration of the lowest discount factor and the highest discount factor to be 50 years (200 quarters). These assumptions yield the following transition matrix[5]:

$$\Upsilon = \begin{pmatrix} 0.9950 & 0.005 & 0.0000 \\ 0.0007 & 0.9979 & 0.0014 \\ 0.0000 & 0.0050 & 0.9950 \end{pmatrix}$$

The conditions that we have described so far specify 10 parameters so 10 targets are needed. When we account for the Gini index and the capital-output ratio, we need 8 additional targets. Given our calibration strategy, those 8 targets would be 8 points from the U.S. wealth distribution. In practice, instead of targeting 8 specific points, we searched for a set of parameters so that the wealth distribution in the model economy is as similar as possible to its U.S. counterpart.

As shown in Table 1.1 this calibration does a fairly good job at approximating the shape and the skewness in the U.S. wealth distribution[6] and yields a Gini index

[5]For further details on the calibration of this matrix, see appendix 2.

[6]The data we report on the U.S. distribution comes from Krusell and Smith (1998) and Budria-Rodriguez, Diaz-Gimenez, Quadrini and Rios-Rull (2002).

Table 1.1: Distribution of wealth: Benchmark model and data

	Held by Top				Gini
Source	1%	5%	10%	20%	
Benchmark Model	22	52	71	89	.82
Data	30	51	64	79	.79
	Held by Bottom				
Source	20%	40%	60%	80%	
Benchmark Model	1	2	4	11	
Data	0	1	6	18	

of 0.82 and a capital-output ratio of 10.6[7].

1.5.3 Labor market processes

For the sake of simplicity our calibration of the aggregate shock and the labor market process follows Krusell and Smith (1998). The process for z is set so that the average duration of good and bad times is 8 quarters. Therefore, the transition matrix η for aggregate state changes is defined by:

$$\eta = \begin{pmatrix} 0.8750 & 0.1250 \\ 0.1250 & 0.8750 \end{pmatrix}$$

The average duration of an unemployment spell is 1.5 quarters in good times and 2.5 quarters in bad times. We also set the unemployment rate accordingly: in good periods it is 4% and in bad periods it is 10%. These assumptions enable us to define

[7]The value of the capital-output ratio can change with the definition of capital. Here we adopt the definition in Quadrini (2000). Thus aggregate capital results from the aggregation of plant and equipment, inventories, land at market value, and residential structures. This definition is close to the findings of Prescott (1986) and is also used for instance in Floden and Linde (2001). This yields a capital-output ratio of 2.65 on an annual basis that we convert to its quarterly equivalent of 10.6.

the transition matrixes for labor market status for each aggregate state change: Π^{gg} for a transition from a good period to a good period, Π^{bb} for a transition from a bad period to a bad period, Π^{gb} for a transition from a good period to a bad period and Π^{bg} for a transition from a bad period to a good period[8]:

$$\Pi^{bb} = \begin{pmatrix} 0.6000 & 0.4000 \\ 0.0445 & 0.9555 \end{pmatrix} \quad \Pi^{bg} = \begin{pmatrix} 0.2500 & 0.7500 \\ 0.0167 & 0.9833 \end{pmatrix}$$

$$\Pi^{gb} = \begin{pmatrix} 0.7500 & 0.2500 \\ 0.0729 & 0.9271 \end{pmatrix} \quad \Pi^{gg} = \begin{pmatrix} 0.3333 & 0.6667 \\ 0.0278 & 0.9722 \end{pmatrix}$$

Finally the joint transition matrix Π for labor market statuses and aggregate states can be defined as:

$$\Pi = \begin{pmatrix} \eta_{bb}\Pi^{bb} & \eta_{bg}\Pi^{bg} \\ \eta_{gb}\Pi^{gb} & \eta_{gg}\Pi^{gg} \end{pmatrix} = \begin{pmatrix} 0.5250 & 0.3500 & 0.0313 & 0.0938 \\ 0.0388 & 0.8361 & 0.0021 & 0.1229 \\ 0.0938 & 0.0313 & 0.2916 & 0.5833 \\ 0.0911 & 0.1158 & 0.0243 & 0.8507 \end{pmatrix}$$

1.5.4 Government

We fix the ratio of government purchases to GDP to 0.217. The debt over GDP ratio, noted b is set to the quarterly value of $\frac{8}{3}$ which is equivalent to an annual

[8]Further details on this calibration can be found in Appendix 2.

value of $\frac{2}{3}$. Those values are the observed ratios in the U.S. as reported by Aiyagari and McGrattan (1998).

2 Results

We now present the results obtained with our benchmark economy. A first section reports the aggregate behavior of the model. A second examines the long-run welfare effects of public debt in an aggregate fluctuations setting. In a third, we move on to the business cycle and distributional effects of public debt. In the next to last section we compare the benchmark economy to simpler idiosyncratic risk only models. Finally, we explore an alternative calibration that changes the labor market process.

2.1 Public debt in an aggregate fluctuations setting

We start by discussing the aggregate behavior of our benchmark model. Our computations are reported in Figure 1.1. Increasing the level of public debt increases the supply of safe assets in the economy. Consequently, the before tax interest rate increases. Because the repayment of debt interests is higher, the income tax rate increases. Nevertheless, the after tax interest rate unambiguously increases. In turn, public debt has a crowding out effect on private capital: higher levels of debt decrease the aggregate amount of private capital in the economy. The crowding out of capital induces the observed decline in output[9].

However, the decline in physical capital is smaller than the increase in public debt. The increase in the after-tax interest rate reduces the gap between the after-tax interest rate and the rate of time preference. The cost of postponing consumption to build up a buffer stock of saving is then reduced. Households choose to hold more assets at the steady-state equilibrium. That is why the overall wealth level A, which is the combination of private capital and public debt, is higher.

[9]of physical capital, the increase in wealth would have been the same as the increase in public debt. The steady state consumption would have been higher.

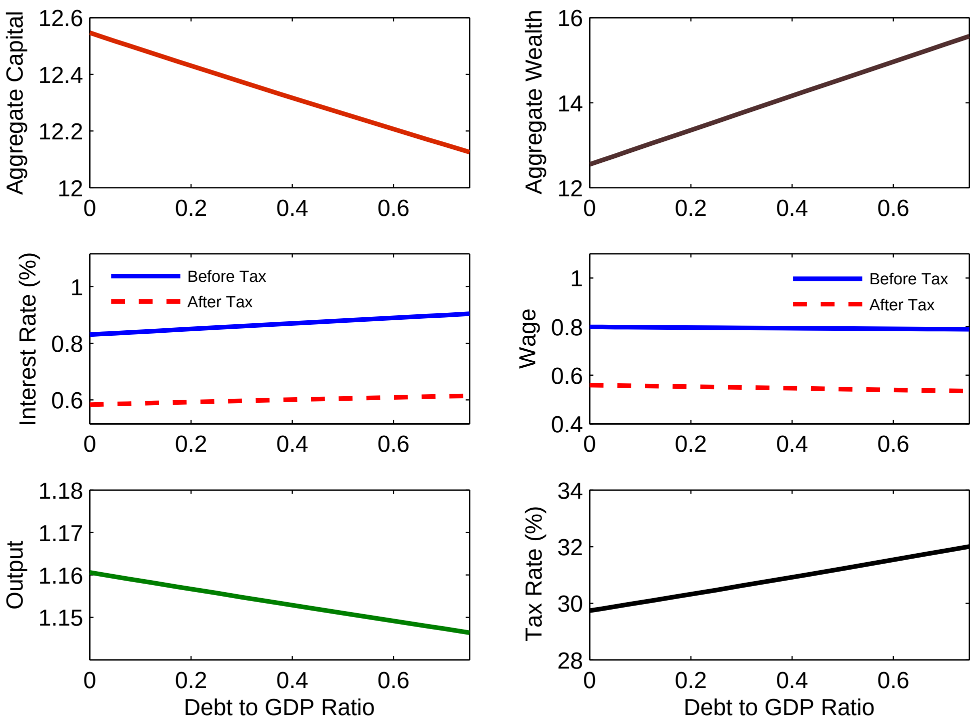

Figure 1.1: Aggregate behavior of the benchmark model

2.2 Welfare analysis and optimal level of debt

The welfare analysis we conduct below apply to the long run optimal level of public debt with aggregate fluctuations. As there is no formal steady-state in a model with aggregate fluctuations, we consider for our welfare analysis the values of aggregate variables averaged over long periods of time for a given debt to GDP ratio[10]. We define the optimal level of public debt as the debt over GDP ratio that maximizes the traditional utilitarian welfare criterion μ.

As explained in Lucas (1987) and Mukoyama and Sahin (2006), this criterion measures the amount of consumption that one would have to remove or add in order to make the agent indifferent between the benchmark debt over GDP ratio and some other level of public debt. It verifies:

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty}\left(\prod_{j=0}^{t}\beta_j\right)\log((1+\mu)c_t^{bench.})\right]=E_0\left[\sum_{t=0}^{\infty}\left(\prod_{j=0}^{t}\beta_j\right)\log(c_t)\right]$$

with $\left\{c_t^{bench.}\right\}_{t=0}^{\infty}$ the consumption stream in the benchmark model when the debt over GDP ratio is equal to $\frac{8}{3}$. $\{c_t\}_{t=0}^{\infty}$ is the consumption stream when the debt over GDP ratio is set to some other level than the benchmark level. For logarithmic utility we can show that:

$$\mu=\exp\left(\left[V-V^{bench.}\right]/S\right)-1,$$

[10]More details can be found in Appendix 1.

where $V^{bench.} = E_0\left[\sum_{t=0}^{\infty}\left(\prod_{j=0}^{t}\beta_j\right)\log(c_t^{bench.})\right]$, $V = E_0\left[\sum_{t=0}^{\infty}\left(\prod_{j=0}^{t}\beta_j\right)\log(c_t)\right]$ and $S = \mathbb{E}_0\left[\sum_{t=0}^{\infty}\left(\prod_{j=0}^{t}\beta_j\right)\right]$.

The result of the introduction of public debt on welfare is *a priori* undetermined because of two opposing effects. The first effect is a *crowding out effect*. The crowding out of physical capital clearly reduces consumption and then welfare. Moreover the increase in the income tax rate tends to amplify the negative impact of public debt on welfare. The second effect is a *cost of precautionary saving effect*: the increase in the after-tax interest rate makes it less costly to accumulate precautionary saving in order to smooth consumption as the interest rate gets closer to the time preference rate. This second effect is welfare enhancing[11]. It is difficult to predict which effect overcomes the other analytically.

Figure 1.2 depicts the long-run optimal level of debt in the benchmark economy. In a setting embedding aggregate risk and calibrated on the U.S. economy, the optimal public debt level is 5% of output on an annual basis. Aggregate fluctuations, idiosyncratic risk and credit constraints lead agents to engage in precautionary saving in order to smooth consumption. Without public debt, the cost of precautionary saving is higher because when households accumulate, the interest rate lowers. Any level of public debt, raises the interest rate and reduces the cost of precautionary saving. As a result welfare is enhanced. However any level of public debt also crowds

[11] Woodford (1990) argues that welfare can be enhanced if the interest rates are kept high enough, that is, closer to time preference rates in a liquidity-constrained economy. We find the same effect here.

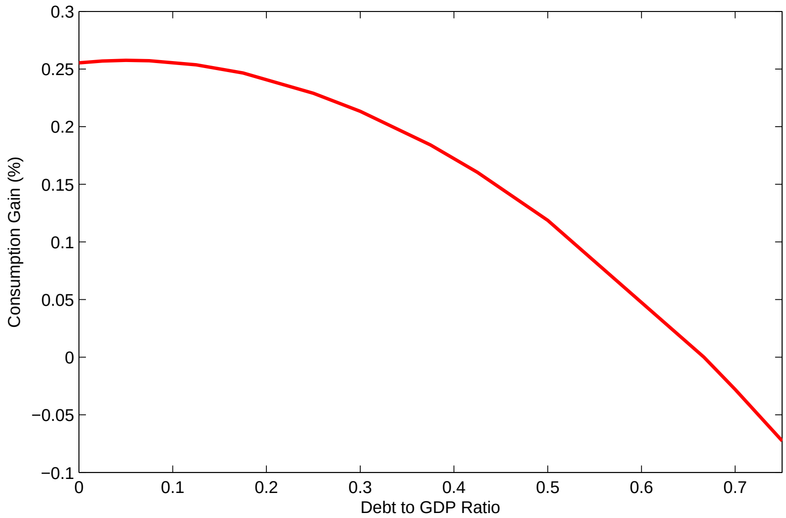

Figure 1.2: Welfare gain versus debt/GDP ratio in the benchmark model

out private capital and reduces welfare.

Here, a lower level of debt than the benchmark level increases welfare out of reducing the crowding out of private capital. At the same time, a lower level of debt than the benchmark level decreases welfare out of increasing the cost of precautionary saving. As long as the first effect dominates the second, a lower level of debt is optimal. At the optimal level, one effect exactly balances the other. For a debt over GDP ratio smaller than 5%, the consumption loss out of increasing the cost of precautionary saving is higher than the consumption gain out of reducing the crowding out of private capital.

As illustrated in Table 1.2, going from the benchmark level of debt to the optimal level of debt is welfare enhancing. The consumption gain of being at the optimal

Table 1.2: Consumption gain (%) of going to the optimal level of debt in the benchmark economy

	Business Cycle		
Population	Average	Recessions	Booms
All	0.257	0.267	0.247
Bottom 10%	0.885	0.901	0.868
Top 10%	-10.401	-10.535	-10.264

level of debt instead of the benchmark level is 0.257%. On the contrary, it is welfare decreasing to go to a higher level of public debt than the benchmark level. In an economy where the level of debt is 75%, the consumption loss not to be at the benchmark level (resp. optimal level) would be on average 0.072% (resp. 0.329%). These results suggest that the role the interest rate plays in reducing the cost of precautionary saving is central to understand why agents settle for the optimal level of public debt.

If we block any price movements by considering a price fixed small open economy, it appears that there is no consumption gains or losses out of the *cost of precautionary saving effect*. In the case where all prices are set to their average benchmark level for any level of public debt, there is no consumption loss out of increasing the cost of precautionary saving for a lower level of public debt than the benchmark level. In this case, there are only consumption gains out of reducing the crowding out of private capital.

2.3 Business cycles and distributional effects

In this section we move on to the effects of public debt along the cycle and across the distribution of the population. To generate statistics along the cycle, we compute separately the average values of our aggregate variables either when the economy is in boom or when it is in recession over the whole sequence of simulation. The last two columns of Table 1.2 illustrate the consumption gain or loss of being at the optimal level of debt instead of the benchmark level. In recession, the consumption gain of being at the optimal level of debt is higher than on average. On the contrary, in expansion this consumption gain is lower. This is due to a *price fluctuation effect*: in recessions, the cost of postponing consumption to build up a buffer stock of savings is higher because the interest rate is lower as shown in Table 1.3. In expansion, we have the opposite: interest rates are higher and the cost of precautionary saving is lower.

The fluctuation of prices is not the only relevant effect we need to account for to explain the optimal level of public debt. When we take aggregate risk into consideration, an *employment fluctuation effect* modifies the precautionary saving motive. Along the cycle, the unemployment rate and duration increase in recessions and reduce in booms. In recessions, the precautionary motive becomes stronger: employed agents face a higher risk of loosing their job and unemployed agents find it more difficult to find a job. In recessions agents want to save more for precautionary motives because of the *employment fluctuation effect* but at the same time the *price* *fluctuation effect* raises the cost of saving. Thus public debt helps to reduce the cost of precautionary saving in recessions. In booms, agents want to save less because of the *employment fluctuation effect*. At the same time the *price fluctuation effect* makes it less costly to save. Thus public debt is less useful in booms.

Table 1.3: Macro variables along the cycle in the benchmark economy

	Level of debt (% of output)					
	66			5		
Statistics	Booms	Average	Recessions	Booms	Average	Recessions
Agg. Capital	12.23	12.17	12.10	12.58	12.52	12.46
Output	1.18	1.14	1.11	1.20	1.16	1.12
Before Tax Interest Rate (%)	0.99	0.90	0.80	0.93	0.83	0.74
Before Tax Wage	0.7902	0.7900	0.7899	0.7982	0.7981	0.7980

The overall consumption gains or losses of a change in the level of public debt is shared very differently in the population. To show that we decompose the welfare gains of going from the benchmark level of public debt to the optimal level across the population. The last two rows of Table 1.2 show this decomposition. For instance, the row *Bottom* 10% shows the welfare gap between the 10% least fortunate people living in an economy with the benchmark level of public debt and those living in an economy with the optimal level of public debt. This decomposition closely matches a decomposition by wealth levels as the lowest (resp. highest) expected utilities refer to people who have experienced the highest unemployment (resp. employment) spells and who end up with the lowest (resp. highest) level of assets. The poorest agents are better off with a lower level of public debt than the optimal level.

Going from the benchmark level of debt to the optimal debt level leads to an increase in consumption of 0.885% for the poorest 10% of the population. In the meantime, the richest 10% of the population would loose as much as 10.401% of consumption. This is explained by the fact that rich people's income is mainly capital income whereas poor people's income is mostly labor income. Thus when the level of public debt is higher, poor people suffers from the reduction in output caused by higher tax rates, lower wages and crowding out of capital. On the contrary, as interest rates raise with higher level of public debt, rich people are better off. The same type of effects are discussed in Ball and Mankiw (1995) and Floden (2001).

2.4 Optimal level of debt without aggregate fluctuations

In this section we look at setups without aggregate fluctuations and compare them with our benchmark economy. There are several ways of eliminating business cycles[12] to derive a comparable long run idiosyncratic model. We look at three methods.

2.4.1 Imrohoroglu (1989) method

In the spirit of Lucas (1987) we replace the aggregate stochastic process with its conditional mean. We then follow Imrohoroglu (1989) to derive the labor market process. The transition probabilities of this economy are set so that the average rate of unemployment and the average duration of unemployment are the same between this economy and the benchmark economy. All other calibrated parameters are kept to their benchmark values, especially time preference rates and the risk

[12] For a survey, see Barlevy (2004).

aversion parameter[13]. This model is similar to the benchmark model in many ways: higher levels of debt raise interest rates and crowd out private capital, overall wealth increases with debt and taxes are higher. But as the risk faced by agents is lower, agents save less and interest rates are higher in this model.

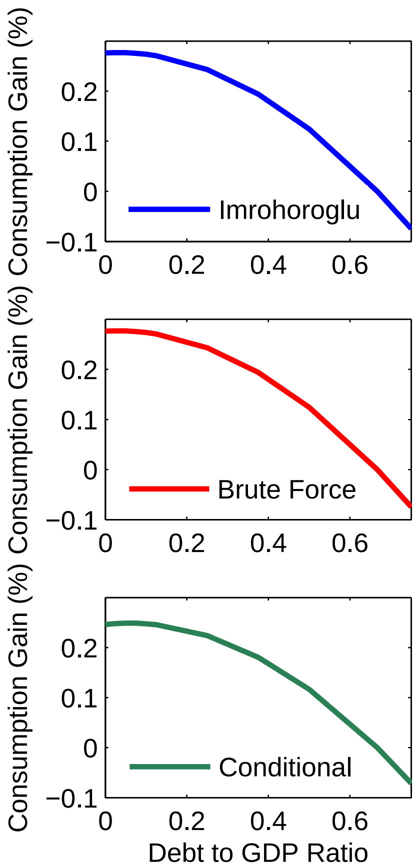

Figure 1.3: Welfare gain versus debt/GDP ratio in idiosyncratic models

Figure 1.3 (top) depicts the welfare profile we find in this idiosyncratic risk model. The optimal level of public debt is 2.5% of output on an annual basis. This level

[13]For greater details on the models without aggregate risk and their calibration, see appendix 3.

is lower than the level we found in the benchmark economy. This is because there is no *price fluctuation effect* and *employment fluctuation effect* in this economy. In the benchmark economy, as the unemployment rate and the unemployment duration increase during recessions, the precautionary motive is stronger. Moreover, in the benchmark economy the interest rate and the physical capital are smaller in recessions. Therefore, it is more costly to save for precautionary motive. That is why the need for public debt is more important in the benchmark economy. In the idiosyncratic model the risk is lower because agents are subject to a less risky labor market and at the same time they no longer face price fluctuations. The precautionary motive is weaker here than in the benchmark model and agents save less. As a consequence interest rates are at a higher level in this economy.

2.4.2 Brute-force averaging

This method is related to the first one above. We again set the aggregate process to its conditional mean. The transition probabilities on the labor markets are generated as follows. We take the exact same transition process as in the benchmark economy and repeat it over very long periods of time. Thus we create a history of transition on the labor market conditional on the aggregate risk faced by agents. In the end of this simulation, we are left with a sequence of aggregate shocks and corresponding labor market situations for individuals. We then brute-force average the sequence of labor market transitions out. To do so, we count the numbers of transitions from unemployed to unemployed, from unemployed to employed, from employed to employed and from employed to unemployed over the entire simulated sequence and then take the mean. As we simulate over very long periods of time, we respect the law of large numbers. After brute-force averaging over the simulated series, we are left with a transition matrix on the labor market.

The matrix we get by brute-force averaging is very similar to the matrix obtained by the Imrohoroglu method above. Figure 1.3 (middle) depicts the welfare profile we find in this idiosyncratic risk model. The optimal level of public debt is 2.5% of output on an annual basis. We can characterize this optimal level in the same way as we did above.

2.4.3 Conditional approach

In this section we consider an alternative method of removing business cycles to obtain a long-term idiosyncratic model by conditioning on the aggregate state. First, as above, we equate the aggregate process to its conditional mean. To obtain the labor market process, we condition on the aggregate state. For instance, we compute the probability of being employed tomorrow conditional on being employed today π_{ee} in the idiosyncratic model as follows:

$$\pi_{ee} = \Pr(z = z_g | s = e) * (\Pi_{bgee} + \Pi_{ggee}) + (1 - \Pr(z = z_g | s = e)) * (\Pi_{gbee} + \Pi_{bbee}),$$

where for instance $\Pi_{ggee} = \Pr(z' = z_g, s' = e | z = z_g, s = e)$.

This means that to find who will remain employed in the next period conditional on being employed today, we consider all occurrences of being employed tomorrow conditional on being employed today in every aggregate state of the benchmark calibration and we weight them accordingly by the probability of being in either good or bad state conditional on being employed. We apply the same method to derive π_{uu}. The remaining probabilities are found by applying the raws summing to unity property of the idiosyncratic transition matrix. This method of removing aggregate risk yields a more persistent labor process than the methods above. Notably, the probability of remaining unemployed tomorrow conditional on being unemployed today is higher. Thus the unemployment rate is not equal to its average value in the benchmark model. Here the unemployment rate is higher and amounts to 7.33% instead of an average value of 7% in the benchmark model.

We find that the optimal level of debt is higher here than with the methods above. The annual optimal debt to GDP ratio is only slightly lower than 5% and is almost identical to the optimal level of debt found is the benchmark model with aggregate risk. This result is depicted in Figure 1.3 (bottom). It is straightforward to see why the optimal level of debt is higher with this method. As both the persistence of the labor market process and the unemployment rate are higher, public debt plays a greater role in reducing the cost of precautionary saving here than with the methods above. We illustrate this in the next section with the benchmark economy.

2.5 Higher unemployment rate and longer unemployment spells

Our benchmark calibration reflects the behavior of the U.S. labor market. In this section we consider an alternative calibration of the labor market, that is a step towards reproducing European labor market features although we do not take into consideration or reproduce the employment benefit system found in Europe. We follow the methodology used in Algan and Allais (2004) and the data set of Blanchard and Wolfer (2000). We only modify the labor market features and leave the rest of the calibration unchanged. We fix the unemployment rate to be 13% in recessions and 7% in booms. We also set the duration of an unemployment spell to be 6 quarters in recessions and 4 quarters in booms. We now find the optimal level of debt to be 30% on an annual basis as depicted in Figure 1.4. Longer unemployment spells and higher unemployment rates tend to raise the optimal level of debt.

This calibration purposely strengthens the *employment fluctuation effect*: unemployed agents have a harder time finding a job in this economy when compared to the benchmark economy and employed agents face a higher risk of loosing their jobs. In recessions, this is amplified. The precautionary motive is stronger here than in the benchmark economy. The harder it is for households to smooth consumption and the higher is the need for public debt. Here a higher level of public debt is needed to help households effectively smooth consumption.

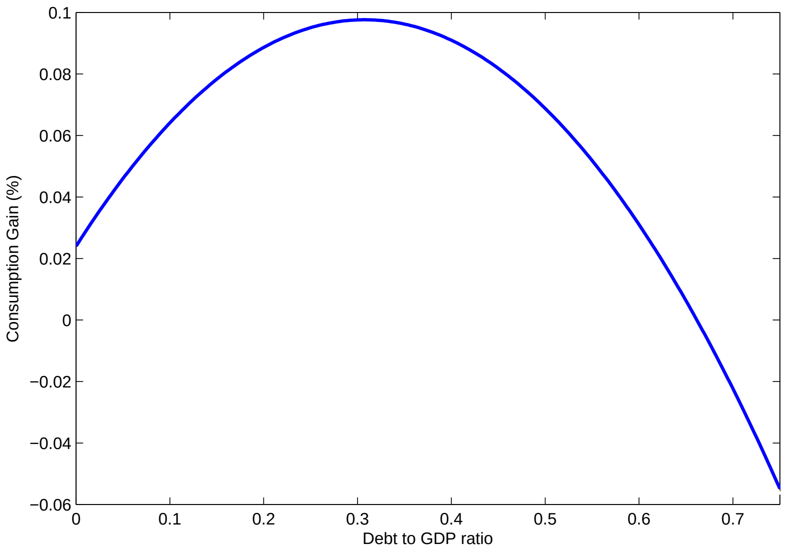

Figure 1.4: Welfare gain versus debt/GDP ratio in the model with higher unemployment rate and longer unemployment spells.

3 Conclusion

This chapter reconsidered the optimal level of public debt in an environment of aggregate fluctuations. Our benchmark model calibrated on the U.S. economy finds that a positive public debt level of 5% of output on an annual basis is optimal. This level is generally higher than in an economy without aggregate fluctuations. Our benchmark economy shows that in an aggregate fluctuations setting, households are subject to an *employment fluctuation effect* and to a *price fluctuation effect*. These effects make the precautionary saving motive stronger and at the same time the cost of saving higher.

Our results suggest that a higher supply of safe assets induced by higher levels of public debt tends to raise the interest rate. This helps households to reduce the cost of precautionary saving and make smoothing of consumption easier. As a result there are welfare gains in the economy. However for the poorest agents, higher levels of debt are not optimal. Poor agents rely mainly on labor income and are dependent on higher wages and lower taxes. Public debt decreases wages and increases tax level thus poor agents prefer lower levels of public debt. We also emphasize that longer unemployment spells tend to raise the optimal level of public debt.

Chapter 2

Uninsured entrepreneurial risk and public debt policies

1 Introduction

Since Aiyagari and McGrattan (1998), it is known that public debt has opposing effects on an economy where agents bear uninsurable idiosyncratic risk on their labor productivity. On the one hand, crowding out of private capital and rising taxes have adverse effects on welfare. On the other hand, higher interest rates reduce the cost of precautionary saving what helps improve welfare. In an economy calibrated on the U.S., Aiyagari and McGrattan (1998) find that the latter effect dominates the others for positive levels of public debt and show that a debt over GDP ratio of 2/3 is optimal.

This result relies on the strong saving behavior of workers in the economy as it is their only means of insuring against idiosyncratic risk. Although precautionary saving by workers is not disputed in the literature, empirical evidence suggests that there is a class of agents, made of entrepreneurial households, that have a strong influence on aggregate saving and output. Entrepreneurs are a small fraction of the economy, around 7.5% as reported by Cagetti and De Nardi (2006), but they earn about 22% of total income and own 40% of total wealth (Quadrini (2000)).

Moreover, their portfolio is poorly diversified and is biased toward their private business assets. Moskowitz and Vissing-Jorgensen (2002) find that 45% of entrepreneurial wealth is invested in their business assets. However, Gentry and Hubbard (2004) argue that entrepreneurs may increase their nonbusiness assets as possible insurance against business risk (this is explored in Covas (2006)) or allocate more saving to liquid assets in anticipation of future business investment needs. But whether it is saving in business assets or nonbusiness assets, findings in Gentry and Hubbard (2004) show that entrepreneurs have much higher saving rates than non-entrepreneurs.

In this chapter, I argue that entrepreneurs should be an important class of agents to consider for public debt policies for at least three reasons. First, although precautionary saving by workers is important, Aiyagari and McGrattan (1998) may have overestimated the optimal level of public debt because they leave workers alone to account for overall saving in the economy when empirical evidence suggests that entrepreneurs are the driving force behind national saving.

Second, entrepreneurs are a distinct class of agents and as such their response to changes in public debt might not be the same as that of workers. On the one hand, as their income heavily relies on entrepreneurial production, they might be more sensible to crowding out coming from higher interest rates. On the other hand, as it is clear from Covas (2006), precautionary saving to insure against idiosyncratic business productivity risk is an important behavior of entrepreneurs that could be helped by higher interest rates.

Last, it is discussed in Ball and Mankiw (1995) and Floden (2001) that gains and losses from higher levels of public debt are shared very differently between poor and rich households. Thus, it is important to consider public debt policies in a framework where the wealth distribution is correctly reproduced and entrepreneurs are key to achieve that goal.

This chapter models a stylized economy with both entrepreneurial and non-entrepreneurial households subject to uninsurable risk. As it is traditional with Bewley-Huggett-Aiyagari class of models, I consider borrowing constraints and market incompleteness. The economy has both a corporate production sector and an entrepreneurial production sector. The government levies proportional taxes and issues public debt in order to finance public consumption. The benchmark calibration parameters are chosen to make comparison easy with the standard results of Aiyagari and McGrattan (1998). This setup yields a negative steady state level of optimal public debt and matches the U.S. wealth distribution and wealth Gini.

The intuition as to why the optimal level of public debt is lower than in Aiyagari and McGrattan (1998) is the following. In this model, workers are not responsible for all of total saving in the economy as an important part of total saving is due to entrepreneurs. For a given level of the capital-output ratio, introducing some percentage of entrepreneurs in the economy dramatically reduces the accumulation behavior of workers as it is compensated by the higher propensity to save of entrepreneurs. When public debt increases, because workers accumulate less, they are more concerned by the adverse effects of crowding out and rising taxes than the beneficial effect of the reduced cost on precautionary saving. Eventually, workers settle for a lower level of optimal public debt in this economy.

However, the behavior of entrepreneurs is very different. Entrepreneurial households are better off with a level of debt higher than the level found in Aiyagari and McGrattan (1998). This is because, in the presence of uninsurable production risk, entrepreneurs always tend to invest in the non-risky asset as a means of insurance. Still, in general equilibrium, this type of precautionary saving lowers the interest rate and the attractiveness of the risky investment is increased. Thus, a larger amount of public debt keeps interest rates at a higher level and helps entrepreneurs in their precautionary saving behavior by reducing the cost of postponing consumption. Entrepreneurs go with higher levels of debt until the beneficial effect of reduced cost on precautionary saving is lower than crowding out of private capital and higher taxes. Because they endure substantial levels of risk, this happens for a high level of public debt. In the end, the optimal level of public debt in the economy is explained by the two opposing effects of debt on workers and entrepreneurs. Because workers are in larger proportions in the economy, the optimal level of public debt is negative and amounts to -110% at the steady state.

The rest of the chapter is organized as follows. Next section describes the benchmark economy. Section 3 details the results. The last section concludes.

2 An Entrepreneurial Economy with Public Debt

The economy is populated by a continuum of infinitely lived households of measure one that can be either workers or entrepreneurs. A fixed fraction of the population are workers. They do not have access to the risky entrepreneurial technology, supply labor elastically to a corporate sector and face uninsurable income risk à la Aiyagari (1994). The remaining fraction of the population are entrepreneurs. They produce using a risky technology and are self-employed.

2.1 The production sector

In this economy one consumption good is produced by two production sectors. One sector is characterized by entrepreneurs that operate their own technology. The other is a corporate sector.

2.1.1 Entrepreneurial sector

Entrepreneurs operate a small business. The risky technology employed by entrepreneurs is represented by[1]:

$$Y_t^e = \theta_t f(k_t)$$

k_t is the capital stock in the risky investment, θ_t denotes productivity. θ_t follows a first-order Markov process. Capital depreciates at a fixed rate δ.

[1]This technology exhibits diminishing returns to scale.

2.1.2 Corporate sector

In the corporate sector we assume that there is a continuum of firms which have a neoclassical production technology and behave competitively in product and factor markets. The corporate output is given by[2]:

$$Y_t^c = F(K_t, Z_t L_t)$$

There is no aggregate risk. K is aggregate capital used in the corporate sector, L the detrended non-entrepreneurial aggregate labor supply and Z the labor productivity. Z grows at the exogenous rate g, so we can write that $Z_t = (1+g)^t$, given that the initial level of labor productivity is set to unity. Capital depreciates at the same constant rate δ as in the entrepreneurial sector. Input markets are competitive and the wage rate w and the interest rate r verify:

$$r + \delta = F_1(K_t, Z_t L_t)$$

$$w = Z_t F_2(K_t, Z_t L_t)$$

[2]The function F exhibits constant returns to scale with respect to K and N, has positive and strictly diminishing marginal products, and satisfies the Inada conditions.

2.2 Households

Households can either be employed in the corporate sector or self-employed in the entrepreneurial sector.

2.2.1 Entrepreneurial households

Entrepreneurs can either invest in their small business and face an uninsurable idiosyncratic productivity shock or accumulate a safe asset yielding a non-stochastic income. Let k_{t+1} denote the resources allocated to the risky investment. The gross risky investment is given by:

$$i_t = k_{t+1} - (1-\delta)k_t$$

Let a^e_{t+1} denote the resources allocated to the safe investment. The return on this investment is noted r_t in each period. Entrepreneurs can also borrow at the same rate. The borrowing constraint faced by entrepreneurs is noted $\bar{a}^e$. The budget constraint of the entrepreneur is the following:

$$
\begin{aligned}
& c_t + k_{t+1} + a^e_{t+1} = x_t \\
& x_{t+1} = \theta_{t+1} f(k_{t+1}) + (1-\delta)k_{t+1} + Tr + (1+r)a^e_{t+1} - \tau\xi_{t+1} \\
& with \;\; \xi_{t+1} = \theta_{t+1} f(k_{t+1}) - \delta k_{t+1} + r a^e_{t+1}
\end{aligned}
$$

where c_t denotes consumption, Tr lump-sum government transfers and τ a tax rate.

Given that, the recursive formulation of the entrepreneurial household's problem is the following:

$$
\begin{aligned}
& v^e(\theta, x) = \max_{c,k',a^{e\prime}} U^e(c) + \beta \mathbb{E}[v^{e\prime}(\theta', x')|\theta] \qquad (2.1) \\
& subject\ to \\
& c + k' + a^{e\prime} = x \\
& x' = \theta' f(k') + (1-\delta)k' + Tr + (1+r)a^{e\prime} - \tau\xi' \\
& k' \geq 0\ and\ a^{e\prime} \geq \bar{a}^e
\end{aligned}
$$

where $v^e(x, \theta)$ is the optimal value function of the entrepreneur, x the entrepreneurs' current period wealth, U^e an utility function and β the discount factor.

2.2.2 Non-entrepreneurial households

Non-entrepreneurial households supply labor elastically to the corporate sector and face uninsurable labor income risk. These households can accumulate a safe asset. Let a_{t+1}^{ne} denote the resources allocated to this safe asset and r the return on this saving in each period. The borrowing constraint faced by these households is noted $\bar{a}^{ne}$. The budget constraint of the non-entrepreneurial household is the following:

$$
\begin{aligned}
& c_t + a_{t+1}^{ne} = y_t \\
& y_{t+1} = (1-\tau) w e_{t+1} l_{t+1} + Tr + (1 + (1-\tau) r) a_{t+1}^{ne}
\end{aligned}
$$

where c_t denotes consumption, w current period wage, l_t labor supply, Tr lump-sum government transfers and τ a tax rate. The labor efficiency process e_t follows a first-order Markov process.

Given that, the recursive formulation of the non-entrepreneurial household's problem is the following:

$$v^{ne}(e,y) = \max_{c,l,a^{ne\prime}} U^{ne}(c,l) + \beta \mathbb{E}[v^{ne\prime}(e',y')|e] \tag{2.2}$$

subject to

$$c + a^{ne\prime} = y$$

$$y' = (1-\tau)we'l' + Tr + (1 + r(1-\tau))a^{ne\prime}$$

$$a^{ne\prime} \geq \bar{a}^{ne}$$

where $v^{ne}(e,y)$ is the optimal value function of the non-entrepreneurial household, y its current period wealth, U^{ne} an utility function and β the discount factor.

2.3 The government

The government issues public debt and levies taxes to finance public expenses. The revenues of entrepreneurial production, labor and returns on capital assets are taxed proportionally at an identical rate τ. The government's budget constraint verifies

$$G + Tr + rB = B' - B + T$$

G is the level of public expenses, Tr is the level of the lump-sum transfers to the households, B the level of public debt and T tax revenues. We assume that public expenses, lump-sum transfers, and public debt are a constant fraction of GDP in every period. We note γ the ratio of public expenses to GDP, φ the ratio of lump-sum transfers to GDP and b the ratio of public debt to GDP. Thus the tax rate has the following simple expression at the steady state:

$$\tau = \frac{\gamma + (r-g)b + \varphi}{1 - \delta\bar{k} + rb} \tag{2.3}$$

where $\bar{k}$ is the capital-output ratio in the economy. A accounts for total wealth in the economy and thus includes physical capital and public debt such that:

$$A = K + B$$

Finally, we detrend all relevant variables by the exogenous growth rate g in order to stationarize the problem. Thus the detrended value of a variable x is noted $\hat{x}$ and verifies: $\hat{x}_t = \frac{x_t}{(1+g)^t}$.

2.4 Equilibrium

The recursive detrended steady state equilibrium of this economy is a value function for the entrepreneur: $\hat{v}^e(\theta, \hat{x})$ and for the non-entrepreneur: $\hat{v}^{ne}(e, \hat{y})$, policy functions for the entrepreneur: $\hat{k}(\theta, \hat{x}), \hat{a}^e(\theta, \hat{x}), \hat{c}^e(\theta, \hat{x})$ and for the non-entrepreneur: $\hat{a}^{ne}(e, \hat{y}), \hat{c}^{ne}(e, \hat{y})$, factor prices $(r, \hat{w})$, capital and labor demand from the corporate sector $(\hat{K}, L)$; a constant cross sectional distribution of entrepreneurs' characteristics $\Gamma^e(\theta, \hat{x})$ of mass χ; a constant cross sectional distribution of non-entrepreneurs' characteristics $\Gamma^{ne}(e, \hat{y})$ of mass $(1 - \chi)$; a tax rate τ such that:

1. Given r and τ, the entrepreneur's policy function solves the entrepreneur's decision problem (2.1)

2. Given r, $\hat{w}$ and τ, the non-entrepreneur's policy function solve the non-entrepreneur's decision problem (2.2)

3. Aggregate capital $\hat{K}$, aggregate labor L and aggregate entrepreneurial capital $\hat{K}^e$ are given by:

$$\hat{K} + \hat{B} = \sum_{\theta \in \Theta} \int \hat{a}^e(\theta, \hat{x}) d\Gamma^e(\theta, \hat{x}) + \sum_{e \in \mathrm{E}} \int \hat{a}^{ne}(e, \hat{y}) d\Gamma^{ne}(e, \hat{y})$$
$$L = \sum_{e \in \mathrm{E}} \int el(e, \hat{y}) d\Gamma^{ne}(e, \hat{y})$$
$$\hat{K}^e = \sum_{\theta \in \Theta} \int \hat{k}(\theta, \hat{x}) d\Gamma^e(\theta, \hat{x})$$

4. Given $\hat{K}$ and L the factor prices are:

$$r = F_{\hat{K}}(\hat{K}, ZL) - \delta$$

$$\hat{w} = F_L(\hat{K}, ZL)$$

5. Given the policy functions of entrepreneurs and workers, the probability mea-

sure of entrepreneurs Γ^e and non-entrepreneurs Γ^{ne} are invariant.

6. Government budget constraint holds.

2.5 Calibration

For the sake of comparison, the model economy is calibrated to match certain observations in the U.S. data. We let one period in the model be one year in the data. We divide our parameters in two sets. The first set of parameters can be estimated from the data. To keep comparison with Aiyagari and McGrattan (1998), most of the parameters in this set are taken from their paper. Table 2.1 sums up the parameters in this set. The second set of parameters is used to match relevant moments of the data. Table 2.2 sums up the latter.

2.5.1 Technology

We choose the corporate production function to be Cobb-Douglas:

$$Y_t^c = F(K_t, Z_t L_t) = K_t^{\alpha}(Z_t L_t)^{1-\alpha} \quad 0 < \alpha < 1$$

Capital share of output α is set to 0.3 as in Aiyagari and McGrattan (1998). Capital depreciates at a rate δ that we set to 0.075.

The entrepreneur's risky technology is given by:

$$Y_t^e = \theta_t k^\nu$$

The entrepreneurial productivity process follows a two-states first-order Markov process. The choice of parameters ν, $\theta \in \Theta$ and the transition probabilities for the Markov process are discussed later on. The fraction of entrepreneurial households χ is fixed to 0.0755% as in Cagetti and De Nardi (2006). The economy grows at the exogenous rate g fixed to 0.0185 as in Aiyagari and McGrattan (1998).

2.5.2 Households

The utility function of the entrepreneurial household is of the CRRA type and writes:

$$U^e(c_t) = \frac{c_t^{1-\mu}}{1-\mu}$$

The non-entrepreneurial household values leisure and has an utility of the following form:

$$U^{ne}(c_t, l_t) = \frac{(c_t^\eta l_t^{1-\eta})^{1-\mu}}{1-\mu}$$

The risk aversion parameter μ is set to 1.5 and the parameter controlling labor elasticity η is set to 0.328 as in Aiyagari and McGrattan (1998). We suppose that the non-entrepreneurial household can not borrow so that $\bar{a}^{ne}$ is 0. Entrepreneurs can borrow and their borrowing constraint $\bar{a}^e$ is set to -4.0[3].

The labor efficiency follows a first-order autoregressive process:

$$e_{t+1} = \rho^{ne} e_t + \varepsilon^{ne}_{t+1}$$

For comparison, we follow Aiyagari and McGrattan (1998) and set ρ^{ne} to 0.6 and ε^{ne} to 0.24. This process is approximated by a seven states discrete process using the methodology of Tauchen and Hussey (1991).

2.5.3 Government

We assume that the ratio of government purchases to GDP γ is 0.217 and the ratio of lump-sum transfers to GDP φ is 0.082 as specified in Aiyagari and McGrattan (1998). In our benchmark economy, the debt over GDP ratio, noted b, is initially set to $\frac{2}{3}$, as it is the average level reported by Aiyagari and McGrattan (1998) in the U.S. for the postwar period.

2.5.4 Reproducing data moments

After the definition of the first set of parameters, we have to choose the remaining six parameters so as to closely match our target economy. The first calibration target is the capital-output ratio. The value of the capital-output ratio varies with

[3] Huggett (1993) suggests a credit limit of one year's average endowment. In the data, individuals can borrow much more than that. The value chosen here is roughly three times the annual net income of an average entrepreneur. I conduct robustness test on this value later on.

Table 2.1:

Parameter	Value	Source
α	0.3	Aiyagari and McGrattan (1998)
δ	0.075	Aiyagari and McGrattan (1998)
χ	0.0755	Cagetti and De Nardi (2006)
g	0.0185	Aiyagari and McGrattan (1998)
b	$\frac{2}{3}$	Aiyagari and McGrattan (1998)
μ	1.5	Aiyagari and McGrattan (1998)
η	0.328	Aiyagari and McGrattan (1998)
$\bar{a}^{ne}$	0	Aiyagari and McGrattan (1998)
$\bar{a}^{e}$	-4	See text and robustness
ρ^{ne}	0.9	Aiyagari and McGrattan (1998)
ε^{ne}	0.21	Aiyagari and McGrattan (1998)
γ	0.217	Aiyagari and McGrattan (1998)
φ	0.082	Aiyagari and McGrattan (1998)

the definition of capital and can range from 2.0 to over 3.0. Here we target a value of 2.5 for the sake of comparison with Aiyagari and McGrattan (1998).

Next we have to match the U.S. wealth Gini and the U.S. wealth distribution as reported by Quadrini (2000). Thus we set the remaining six parameters as follows. We choose the discount factor to be 0.978. The entrepreneurial ability θ can take only two values. The low value θ_{low} is 0.3, the high value θ_{high} is 1.11. This implies that the transition matrix π for entrepreneurial ability is a 2-by-2 matrix with each row adding to unity. Thus this gives two parameters to pin down. We fix $\pi_{11} = P(\theta' = \theta_{low} | \theta = \theta_{low})$ to 0.988 and $\pi_{21} = P(\theta' = \theta_{low} | \theta = \theta_{high})$ to 0.115. Finally we have to choose a value for ν, the degree of decreasing returns to scale of the entrepreneurial technology. We fix this parameter to 0.6. Thus we have six calibrated parameters to match six observed statistics: the capital output-ratio, the wealth

Gini, and the top four percentile of the distribution of wealth. The results of this calibration and comparison with U.S. data and closely related models are reported in Table 2.3 and discussed in the results section.

Table 2.2:

Parameter	Value
β	0.978
θ_{low}	0.3
θ_{high}	1.11
π_{11}	0.988
π_{21}	0.115
ν	0.6

3 Results

This section presents the results obtained with the benchmark economy. A first section reports the overall behavior of the model. Next, I report wealth distribution statistics for the benchmark economy and several similar models. Then I examine the long-run welfare effects of public debt in this stylized economy. Eventually, I conduct experiments to assess the robustness of the model to parameter values.

3.1 Public debt in a stylized setting with entrepreneurs

The benchmark model yields an equilibrium interest rate of 5.9%[4], on average entrepreneurs invest 45% of their wealth in business equity (Moskowitz and Vissing-Jorgensen (2002) find an empirical equivalent of 45%) and they own 49% of total wealth (Quadrini (2000) find an empirical equivalent of 40%).

Figure 2.1 reports general properties of the benchmark model. Increasing the level of public debt raises the supply of safe assets in the economy. Consequently, the after tax and before tax interest rates increase and the wage rate decreases as shown in the first and second quarter of Figure 2.1. Public debt has a crowding out effect on corporate capital. The increase in the after-tax interest rate reduces the gap between the after-tax interest rate and the rate of time preference. The cost of postponing consumption to build up a buffer stock of saving is then reduced. Households choose to hold more assets at the steady state equilibrium. Also, because the repayment of debt interests increases, the income tax rate generally increases. But the tax rate does not change monotonically. This property is similar to the result in Aiyagari and McGrattan (1998). This is due to changes in the tax base. Hours worked steadily decrease because of the diminishing wage rate.

Figure 2.2 reports entrepreneurial saving behavior in the risky and the non-risky assets. The lower half of Figure 2.2 reports entrepreneurial investment in the safe

[4]Aiyagari and McGrattan (1998) set the interest rate to 4.5% through the calibration of the capital-output ratio. Here the immediate link between the capital-output ratio and the interest rate is broken because of the entrepreneurs. Cagetti and De Nardi (2006) find an equilibrium interest rate of 6.5% in a model with both workers and entrepreneurs.

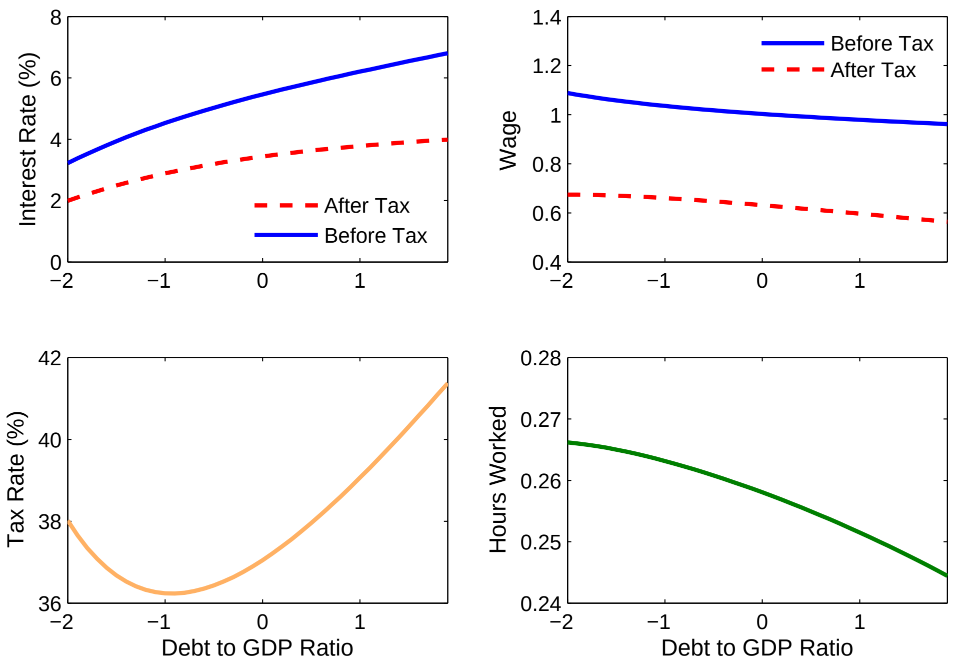

Figure 2.1: Interest rates, wage rates, tax rate and hours worked

asset. There is a steady increase of the safe investment as the debt over GDP ratio rises. As reported by Covas (2006), in the presence of uninsurable production risk, entrepreneurs always prefer investing in the safe asset. However, this behavior tends to decrease the interest rate in general equilibrium and raises the attractiveness of the risky investment. Public debt by increasing the supply of safe assets increases the interest rate. Thus, entrepreneurs accumulate more of the safe investment as the level of debt rises and as the cost of precautionary saving is decreased.

Interestingly, the upper half of Figure 2.2 shows that the accumulation of risky entrepreneurial capital is non-monotonic as the level of debt increases. For low values of the interest rate, raising the level of public debt crowds in entrepreneurial capital and it is crowded out the rest of the time. In his seminal paper, Woodford (1990) shows how higher levels of public debt can crowd in private capital. Woodford (1990) argues that public debt can crowd in private capital when households are liquidity constrained and face interesting risky investment opportunities only at some periods and not all the time. Here, entrepreneurial households can be liquidity constrained and although they face investment opportunities at all times, some opportunities are more interesting than others because of the uninsurable production risk.

Thus, crowding in à la Woodford (1990) can occur in this economy but are completed by other effects. First, note that the profile of the risky investment when debt increases is extremely flat, notably a lot flatter than the safe investment profile. Note also that the tax profile is decreasing for some levels of public debt when entrepreneurial capital is crowded in. For a given increase in public debt, the interest rate rises, entrepreneurs accumulate more safe assets and the financial income of entrepreneurs increases. But as long as the expected return on the risky asset is higher than the return on the safe asset, entrepreneurs can use their higher financial income to invest both in the risky asset and the safe asset and entrepreneurial capital can be crowded in. When the expected return on the risky asset is lower than the return on the safe asset, entrepreneurial capital is crowded out.

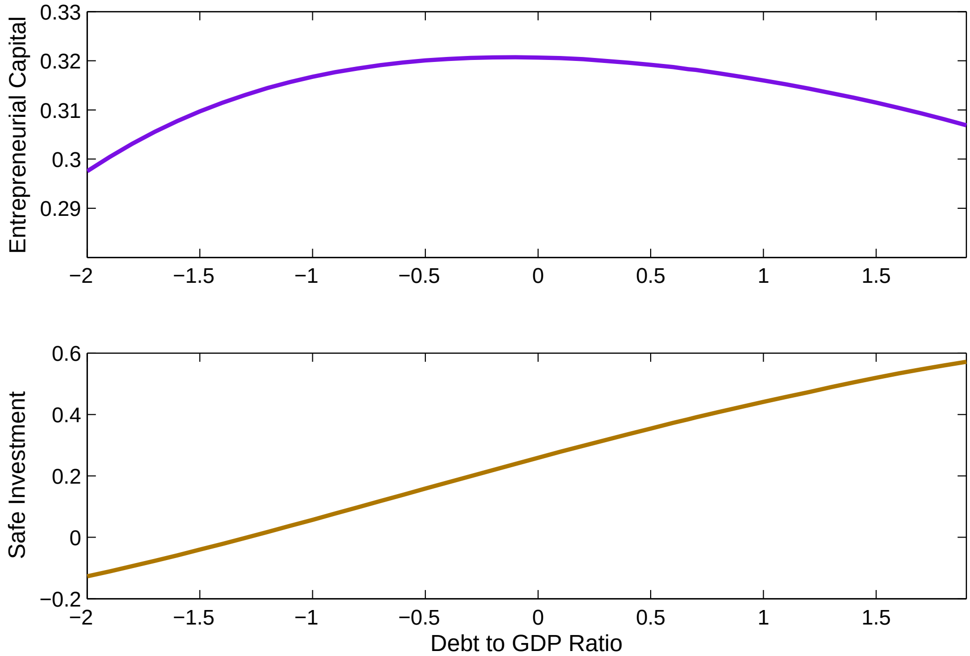

Figure 2.2: Entrepreneurial risky and non-risky assets

3.2 Wealth distribution

As reported in Table 2.3, the benchmark economy yields an improved reproduction of the U.S. wealth distribution and wealth Gini. For the sake of comparison the table shows several related models. First, we reproduce the model in Aiyagari and McGrattan (1998) and present their wealth distribution and Gini. Neither the Gini nor the skewness of the wealth distribution is reproduced by this model. Next we extend this model with the labor productivity process found in Floden (2001), which is more consistent with the results found in the literature of a more persistent income process (see for instance Storesletten et al. (2004)), and we adjust the discount factor to match a capital-output ratio of 2.5. Although the statistics are slightly better, this is still a poor reproduction of the data. Next, we report our benchmark model but without entrepreneurial households. The statistics displayed are similar to those of the model in Aiyagari and McGrattan (1998) as the labor productivity process is the same.

Finally, the next to last row displays the wealth distribution statistics found with the benchmark model where both entrepreneurial and non-entrepreneurial households are taken into account. The statistics are fairly close to the data[5]. The benchmark model reproduces the thick right tail found in the U.S. distribution. Most notably, the top 1% of the population hold 30% of total wealth in the economy and the Gini coefficient is 0.78. Thus, to the best of my knowledge, this model is the best setting to assess the steady state long-run optimal level of public debt with regards to wealth statistics so far. Reproducing the wealth distribution is not neutral when assessing public debt. As it is discussed in Floden (2001) or Ball and Mankiw (1995), public debt mostly benefit wealthy households because they gain from higher return on capital when poorer people suffer from higher taxes and lower wages.

[5] I used as a reference the wealth distribution computed from the Survey of Consumer Finances 1992 as reported by Quadrini (2000).

Table 2.3: U.S. and models wealth distribution comparison

Model	Capital-output ratio	Wealth Gini	Percentage wealth held by top 1%	5%	10%	20%
Aiyagari and McGrattan (1998)	2.5	0.42	4	15	26	44
Floden (2001)* (see text)	2.5	0.63	6	25	41	64
Baseline model without entrepreneurs	2.1	0.43	4	15	27	45
Benchmark model with entrepreneurs	2.5	0.78	30.8	55.5	63.5	75.2
U.S. Data	2.5	0.78	29.5	53.5	66.1	79.5

3.3 Steady State optimal level of public debt

The result of the introduction of public debt on welfare is *a priori* undetermined because of several opposing effects. First, the crowding out of physical capital clearly reduces consumption and then welfare. Moreover the increase in the income tax rate tends to amplify the negative impact of public debt on welfare. Then the increase in the after-tax interest rate makes it less costly to accumulate precautionary saving in order to smooth consumption as the interest rate gets closer to the time preference rate. This last effect is welfare enhancing. It is difficult to predict which effect overcomes the others analytically.

In a stylized setting with both entrepreneurial and non-entrepreneurial households and reproducing key statistics of the U.S. wealth distribution, I find that the annual optimal level of public debt is negative and amounts to -110% of GPD. This is to oppose to the result in Aiyagari and McGrattan (1998) where a positive level of debt of 66% is found. The welfare gain of being at the optimum level instead of the benchmark level amounts to 1.8% of consumption.

The intuition behind this result is the following. Although entrepreneurial house-

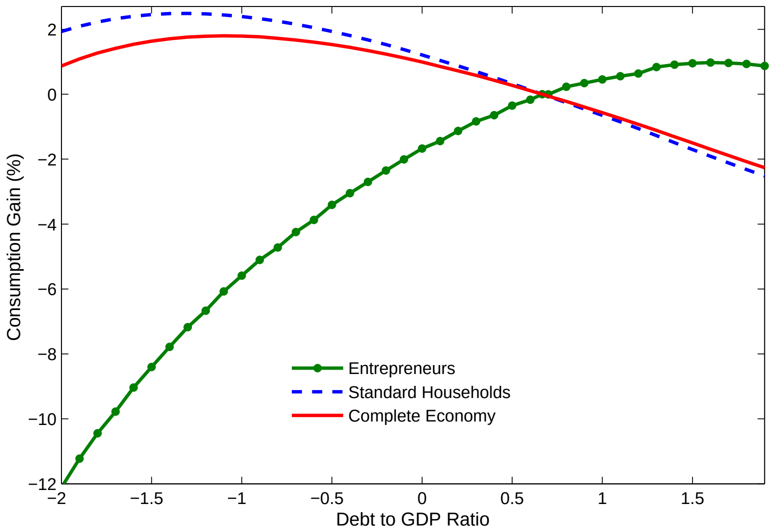

Figure 2.3: Consumption gain versus debt/GDP ratio

holds constitute a small fraction of the population, they have a higher propensity to save. For a given level of public debt and a given set of parameters, the equilibrium capital-output ratio is always higher in an economy where at least a fraction of the population are entrepreneurs. As a result, workers in this model need to accumulate a lot less than in Aiyagari and McGrattan (1998). Accumulation by workers is replaced by the empirically relevant accumulation of entrepreneurs. This is why the economy discounts time by according more importance to the present than in Aiyagari and McGrattan (1998). Thus the discount factor is set to 0.978 in the benchmark model and is much lower than the value in Aiyagari and McGrattan (1998) where this parameter is 0.991.

This adjustment explains an important part of the change in the optimal level of public debt. The discount factor plays a major role in determining how much consumption smoothing an agent needs and thus impacts the balance between crowding out and the reduced precautionary saving cost. The higher the discount rate, the higher the need for consumption smoothing and the higher the optimal level of public debt. My computations show that a setting with only non-entrepreneurial households subject to the same calibration and most notably to the same discount rate as in the benchmark economy yields an optimal level of public debt of −80%.

Scaling the discount factor to the targeted capital-output ratio explains an important part of the adjustment in the optimal level of debt. But interestingly, this level of debt is still lower with entrepreneurs than without even when we control for the adjustment in the discount factor. The exact same economy as in the benchmark model but without entrepreneurs yields an optimal level of public debt of −80%. In Aiyagari and McGrattan (1998) the discount factor is set to 0.991. In the benchmark economy here, the discount factor is 0.978. Thus in an economy that exactly reproduces the optimal level of debt of 66% of Aiyagari and McGrattan (1998) with a discount factor of 0.991, the optimal level of debt becomes −80% if the discount factor is diminished so as to reach its level in the benchmark economy, namely 0.978.

Now, when even a small fraction of the population are entrepreneurs, the optimal level of debt becomes lower than this latter value of −80%. This is the result of two opposing effects. First, as shown by the bulleted line in Figure 2.3, the optimal level of debt for entrepreneurs alone is higher than the benchmark level. In fact, the optimal level of debt for entrepreneurs would be 160% of GDP and entrepreneurs would gain as much as 0.98% of consumption if public debt was at this level instead of the benchmark level. In the presence of uninsurable production risk, entrepreneurs always prefer the non-risky asset. However, in general equilibrium, this type of precautionary saving tends to lower the interest rate and the attractiveness of the risky investment is increased. This result is reported in Covas (2006). Without public debt, precautionary saving would lower interest rates. Here, a positive amount of public debt, keeps interest rates at a higher level and helps entrepreneurs to smooth consumption. Entrepreneurs go for higher levels of public debt until the positive impact of reduced precautionary saving cost is balanced by the adverse effects of increasing taxes and crowding out.

Second, as shown by the dashed line in Figure 2.3, the optimal level of public debt for non-entrepreneurial households alone is -130% and they would gain 2.5% of consumption if public debt was at this level instead of the benchmark level. This is mainly because the tax rate is higher in an economy with entrepreneurs. For a given level of public debt, introducing entrepreneurial households has two general equilibrium effects that can impact the tax rate. On the one hand, the interest rate is lower because the precautionary saving motive of entrepreneurial households is higher than that of their non-entrepreneurial counterparts. As shown by Equation 2.3, the effect of the interest rate on the tax rate is ambiguous. On the other hand, the capital-output ratio is higher in an economy with entrepreneurs. We can see in Equation 2.3 that a higher capital-output ratio unambiguously raises the steady state tax rate. My computations show that this last effect combined with the tax rate increasing effect of the interest rate dominate the tax rate decreasing effect of the interest rate.

Eventually, for non entrepreneurial households, the adverse effects on welfare of higher taxes and crowding out of capital balance the welfare increasing effect of reduced precautionary saving cost for a lower level of public debt when there are even a slight fraction of entrepreneurs in the economy. Here, as they are the most numerous type of households and because the adverse effects of public debt is higher for them than for entrepreneurs, non-entrepreneurs drive the impact on the overall level of welfare in the economy. In the end, as shown by the straight line in Figure 2.3, overall consumption gain is maximum for a level of debt of -110% and the whole economy would gain as much as 1.80% of consumption if public debt was at this level instead of the benchmark level. For this level of public debt, entrepreneurs alone would lose as much as 6.0% of consumption whereas non-entrepreneurs alone would gain as much as 2.4% of consumption.

3.4 Robustness to the credit constraint

In this section, we experiment the effect on the optimal level of public debt of a change in the borrowing constraint of entrepreneurs. In the benchmark economy, this constraint is set to a value that represents roughly three times the annual net income of an average entrepreneur. It is difficult to pin down an exact value for this parameter. Huggett (1993) suggests a credit limit of one year's average endowment. Covas (2006) loosen up this value by one year and reports that in the data individuals can often borrow much more. Generally speaking, Cagetti and De Nardi (2006) argue that the borrowing constraint is related to the level of the entrepreneur's wealth[6]. How much the entrepreneur can borrow is an important aspect that will have an effect on his interaction with public debt. Thus in my experiment, I alternatively loosen and tighten the borrowing constraint of the entrepreneur. The results of this experiment is reported in Figure 2.4.

Figure 2.4: Optimal level of public debt and the borrowing constraint of entrepreneurs

In the benchmark economy, the optimal level of public debt is −110%. If the

[6]For a survey on entrepreneurship and borrowing constraints see Cagetti and De Nardi (2006).

borrowing constraint of entrepreneurs is loosened to a value that is about 1.5 times the benchmark value (this means a borrowing constraint set to -6), the optimal level of public debt is -130%. Symmetrically, if the borrowing constraint is tightened to a value of 0.75 times the benchmark value (this means a borrowing constraint set to -3), the optimal level of debt is -100%. If it tightened even more, to a value of 0.25 times the benchmark value (this means a borrowing constraint set to -1), the optimal level of debt would be slightly over -90%.

The main result of this experiment is that a tighter borrowing constraint produces a higher optimal level of public debt. This is not surprising and is correlated with the results in Covas (2006). A tighter borrowing constraint makes it more difficult for entrepreneurs to smooth consumption to insure against adverse shocks. As the level of risk faced by entrepreneurs is independent of the borrowing constraint, a tighter borrowing constraint gives them a stronger incentive to self-insure. As a result, the tighter the borrowing constraint, the higher is the need of a greater level of public debt for entrepreneurs.

4 Conclusion

I model a stylized economy with entrepreneurial and non-entrepreneurial households in order to assess the optimal level of public debt. The setting of this chapter reproduces key statistics of the U.S. wealth distribution and yields an optimal level of public debt of -110%. This value is lower than the result in Aiyagari and McGrattan (1998). I decompose the effects that produce this level of public debt. First, entrepreneurs always want highly positive levels of public debt. This is because they are subject to idiosyncratic productivity risk that increases their need for precautionary saving.

On the contrary, non-entrepreneurial households are better off with lower levels of debt because part of their accumulation has been transferred to entrepreneurs and because the tax rate is higher in an economy with entrepreneurs. But as entrepreneurs are less numerous, their impact on the overall welfare level in the economy is not as important as non-entrepreneurs. The important qualitative result of this chapter is that entrepreneurs reduce the accumulation behavior of non-entrepreneurs and thus that optimal level of public debt is lower in an economy with entrepreneurs than in an economy without.

Chapter 3

Optimal public debt without commitment[1]

[1]This chapter is a joint work with Audrey Desbonnet and Thomas Weitzenblum

Introduction

The question of choosing public debt over time is intricately connected with that of commitment. When a commitment technology is available, the literature has described how a benevolent government would choose public debt. In most of the cases, there is an initial increase and then the level of public debt is constant for all remaining periods, the long-run level being dependent on the initial level. However, it is arguable that perfect commitment technologies are unavailable to actual governments. Thus, after considering substitutes to commitment[2], a recent strand of the literature has focused explicitly on the case of policies without a commitment technology[3].

Such policies are said to be time-consistent and describe successive forward-looking governments playing a game against each other in time. In an effort to explain why most governments end up with large positive amounts of public debt, Krusell, Martin and Rios-Rull (2006) relax the commitment assumption in a Lucas-Stokey (1983) type framework without capital or any type of risk. They find that there is no strong bias in favor of high government debt and that the dynamics resemble the case of commitment. Evidence from Aiyagari, Marcet, Sargent and Seppala (2002) and Shin (2006) suggest that in the case of commitment market structure and heterogeneity play a crucial role in determining long-run debt.

[2]See for instance Lucas and Stokey (1983) or Chari and Kehoe (1990).

[3]A series of papers explore the effects of time-consistency for public policies. See for instance Krusell (2002), Klein and Rios-Rull (2002) or Klein, Krusell and Rios-Rull (2007).

As it is clear from the stationary debt literature under incomplete markets[4], in the long-run the optimal level of public debt is a trade-off between costs of crowding out of private capital and higher taxes and gains from reduced cost of consumption smoothing. Desbonnet and Weitzenblum (2007) show that the transitional dynamics of such models exhibits much higher optimal level of public debt than the stationary level because of the steep decrease of the tax rate subsequent to the public debt shock. However, in these models the government is not time-consistent.

In this chapter I argue that the assumptions of commitment, market completeness and uncertainty have a strong impact on the level of optimal public debt and on the way it is chosen over time. The empirically relevant feature is that almost all governments end up with large positive amounts of public debt. On the one hand and to the best of my knowledge, models without commitment remain in the simplifying assumptions of complete markets and representative households and are unsuccessful in explaining the observed positive amount of public debt. On the other hand, models with incomplete markets fail to capture the important behavior of choosing debt over time and its implication on the optimal level of public debt.

Thus, this chapter builds the simplest possible setting to relax these assumptions in order to assess the optimal level of public debt. The benchmark setting introduces a time-consistent equilibrium for public debt choice where households face uninsurable risk. In the tradition of Bewley-Huggett-Aiyagari class of models,

[4]See for instance the seminal paper of Aiyagari and McGrattan (1998).

I impose borrowing constraints and market incompleteness. A representative firm produces a single good. The government levies proportional taxes and issues public debt in order to finance public consumption. The political process is the following: at each period, a benevolent planner (or government) has to choose the level of public debt for the future period with a given probability. In the case a choice has to be made, the planner maximizes social welfare in order to set the level of public debt. Otherwise, the current level of debt is maintained.

A simple calibration scheme yields first that the time-consistent level of public debt is quite different from its stationary counterpart. Second, the time-consistent debt to GDP ratio is constantly large and positive. The intuition behind this result is the following. In a time-consistent equilibrium, agents are fully rational and can predict the outcome of future debt choices. Moreover, agents make their own choices knowing their effects on the evolution of the economy. Increasing the level of debt today has the immediate effect of bringing the tax rate down. As it will boost current consumption, this is welfare increasing. But agents know that tomorrow and the consequent periods, they will have to pay higher taxes until a new choice can be made. Similarly a higher level of debt has an effect on prices. The interest rate immediately increases but falls down in the future. Higher interest rates can help agents to reduce the cost of precautionary saving and increase welfare. But as higher levels of debt crowd out private capital, agents have also to endure welfare losses. Agents will choose higher levels of debt until the immediate gains are equal to future losses. Eventually, the benevolent planner will set a level of debt that maximizes total welfare in the economy.

The rest of the chapter is organized as follows. Next section describes the benchmark economy. Section 2 details the results. The last section concludes.

1 The model

The economy is a Bewley-Huggett-Aiyagari type dynamic stochastic general equilibrium model where a benevolent social planner, that we call government, maximizes social welfare in order to adjust the level of public debt. The latter behavior of the government is called policy and describes the political problem of choosing sequentially in time a current period optimal level of public debt. This problem is in essence similar to the one described in Krusell, Quadrini and Rios-Rull (1997). The model consists of two interdependent parts. The competitive equilibrium will be characterized by (i) a law of motion for the economy and (ii) a policy rule. The former computes next period's level of capital, given the current state of the economy. The latter associates, to any state of the economy, the level of debt chosen by the government.

1.1 Competitive equilibrium under a given policy

1.1.1 Technology

We assume that there is a representative firm which has a neoclassical production technology and behaves competitively in product and factor markets. The output is given by:

$$Y = F(K, N)$$

where K is aggregate capital and N aggregate labor used in production. The function F exhibits constant returns to scale with respect to K and N, has positive and strictly diminishing marginal products, and satisfies the Inada conditions. Capital depreciates at a constant rate δ. Since inputs market are competitive, the wage w and the interest rate r verify:

$$r + \delta = F_K(K, N)$$

$$w = F_N(K, N)$$

1.1.2 The government

The government issues public debt and levies taxes to finance public expenses. At each date, the government can adjust the level of public debt. Both the revenues of capital and labor are taxed proportionally at an identical rate τ. The government budget constraint verifies:

$$G + rB + TR = B' - B + T$$

with

$$T = \tau(wN + rA)$$

G is the level of public expenses, B the current level of public debt, T tax revenues and TR a lump sum transfer. A accounts for total wealth in the economy and thus includes physical capital and public debt such that $A = K + B$. The tax rate can be made explicit by writing:

$$\tau = \frac{G + (1 + r)B - B' + TR}{(wN + rA)} \tag{3.1}$$

Note that current debt refers to the amount of debt repaid and not to the amount issued. Thus, at a given time, public debt B is a predetermined variable, while B' represents the debt choice, made at the current period and to be paid back in the next period. The ratios of public expenses and transfers to GDP will be constant in all the simulations.

1.1.3 Households

The economy is populated by a continuum of *ex ante* identical infinitely lived households of unit mass. Their preferences, assumed to be additively separable over time, are summarized by the function V:

$$V = \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t u(c_t) \right\} \tag{3.2}$$

where β is a discount factor. Leisure is not valued.

Each period, households are subject to a productivity shock, which dynamics are modeled as a Markov chain on a finite set S. The Markov chain is fully described by a transition probability matrix $(\pi_{ss'})_{s,s' \in S}$ where $\pi_{ss'}$ represents the probability that next period's productivity shock is s', given that it is s at the current date. This shock is assumed to be uninsurable, and a strict borrowing constraint is imposed upon households such that the amount of financial assets they hold, a, always respects the constraint $a' \geq a_{min}$. Therefore, the current level of financial assets held by a given household will depend on the past realizations of the productivity shock, giving rise to heterogeneity with respect to financial assets as in Huggett (1993) or Aiyagari (1994). The consumption/saving behavior of a household then depends on its individual state (a, s), and on the aggregate state of the economy. The latter affects individual behavior through its impact on the interest rate, the wage and the current and future tax rates.

Our representation of the state space can then be summarized by the following vector: (a, s, Ψ, B, B') where Ψ denotes the density function of households over the state space. It is clear that this vector fully characterizes the current state of the individual. More precisely, the average asset holding writes:

$$A = \sum_{s \in S} \int_{a_{min}}^{a_{max}} a\Psi(a, s)da.$$

The level of aggregate capital is equal to the average asset holding net of the current level of public debt. Given the aggregate capital, we can derive the interest rate and the wage rate. Finally, the current tax rate is obtained from equation (3.1) given the current interest rate and the current and future debt levels.

In addition to these variables that exhaustively describe the current state of a household, expectations regarding future prices and future tax rates have to be taken into account. Precisely, the future interest rate depends on the expected level of capital supply or equivalently, on financial assets net of the future level of debt. Therefore, we need a law of motion describing the future distribution of agents given the current state of the economy. As for the future tax rate, it depends on the future level of debt. In the repeated game that we model here, at each period, a new level of debt B'' will be chosen tomorrow with a probability λ. Otherwise, the level of debt chosen today, B', will remain unchanged in the next period.

At each date, agents anticipate that the level of debt might change or remain constant in the next period. In their expectations of a debt change, agents take future policy choices as given. This means that the expectations regarding future debt levels require the knowledge of a policy rule, which associates the chosen level of debt to any possible aggregate state of the economy. Let us denote by $\Gamma\left(\Psi, B, B'\right)$ the law of motion for the distribution of agents over the state space and by $\Theta\left(\Psi, B\right)$ the policy rule for the future debt choice. The recursive formulation of the household's program then writes:

$$V\left(a, s, \Psi, B, B'\right) = \max_{c,a'} u(c) + \beta E V\left(a', s', \Psi', B', B''\right) \tag{3.3}$$

s.t.

$$a' = (1 + r\left(1 - \tau\left(\Psi, B, B'\right)\right))a + ws\left(1 - \tau\left(\Psi, B, B'\right)\right) + TR$$

$$\Psi' = \Gamma\left(\Psi, B, B'\right)$$

$$B'' = \Theta\left(\Psi', B'\right) \text{ with probability } \lambda$$

$$B'' = B' \text{ with probability } (1 - \lambda)$$

$$r = r(\Psi)$$

$$w = w(\Psi)$$

$$c \geq 0$$

$$a' \geq a_{min}$$

Here, the new level of debt chosen today, B', appears as a state variable, because we allow for deviations in terms of future debt. In other words, this program does not specify how debt is chosen in the current period. This issue is addressed in the next subsection.

1.2 The dynamic game of the benevolent social planner

At each date, we assume that the social planner can choose a new debt level with a given probability λ. In the limit case where λ equals one, the government sets public debt at each period. Although the government can freely choose the current level of debt, this policy lasts only until the next adjustment as no commitment technology is available. Because it has no control over future choices, the government can be seen as playing a game against its future self. As future government choices are exogenous to both the agents and the government, they can be regarded as reaction functions. In the end, the social planner sets today the level of public debt for the subsequent period by maximizing the following utilitarian welfare criterion:

$$B' = \max_{B'} \int_{a_{min}}^{a_{max}} V\left(a, s, \Psi, B, B'\right) \Psi(a) da$$

1.3 The politico-economic equilibrium

The recursive equilibrium is characterized by the vector:

$$\left[a'\left(a, s, \Psi, B, B'\right), V\left(a, s, \Psi, B, B'\right), \Gamma\left(\Psi, B, B'\right), \Theta\left(\Psi, B\right)\right]$$

such that:

1. Given the law of motion $\Gamma(\Psi, B, B')$ and the policy rule $\Theta(\Psi, B')$, $V(a, s, \Psi, B, B')$ is the value function solution to program (3.3) and $a'(a, s, \Psi_t, B_t, B_{t+1})$ the associated saving rule,

2. Given the above value function, the maximization of the utilitarian criterion at each date is consistent with the expected policy rule $\Theta(\Psi, B)$:

$$\forall B, \forall \Psi, \Theta(\Psi, B) = \max_{B'} \int_{a_{min}}^{a_{max}} V(a, s, \Psi, B, B') \Psi(a) da,$$

3. Given the saving rule $a'(a, s, \Psi, B, B')$, and for any state of the economy (Ψ, B), next period's distribution of agents, Ψ', implied by the saving rule, is consistent with the expected law of motion $\Gamma(\Psi, B, B')$.

1.4 Approximate aggregation

Unfortunately, the characterization of the equilibrium above is not tractable because the distribution of agents belongs to a set of infinite dimension. To circumvent this difficulty, we follow the approach pioneered in Den Haan (1996) and Krusell and Smith (1998): we approximate the distribution of agents by the mean value of asset holdings. This choice is motivated by the fact that factor prices determination requires the aggregate stock of capital, which is immediately derived from the average asset holding of households and the current level of debt:

$$K = \sum_{s \in S} \int_{a_{min}}^{a_{max}} a\Psi(a,s)da - B$$

This projection greatly simplifies both the law of motion, that now maps $\mathbb{R}^3$ into $\mathbb{R}$, and the policy choice, that now maps $\mathbb{R}^2$ into $\mathbb{R}$. The recursive program of households can be rewritten as follows:

$$V(a,s,A,B,B') = \max_{c,a'} u(c) + \beta EV(a',s',A',B',B'') \tag{3.4}$$

s.t.

$$a' = (1 + r(1 - \hat{\tau}(A,B,B')))a + ws(1 - \hat{\tau}(A,B,B')) + TR$$

$$A' = \hat{\Gamma}(A,B,B')$$

$$B'' = \hat{\Theta}(A',B') \text{ with probability } \lambda$$

$$B'' = B' \text{ with probability } (1-\lambda)$$

$$r = \hat{r}(A)$$

$$w = \hat{w}(A)$$

$$c \geq 0$$

$$a' \geq a_{min}$$

The modified version of the equilibrium can then be stated as follows:

The recursive equilibrium is characterized by the vector:

$$\left(a'\left(a,s,A,B,B'\right), V\left(a,s,A,B,B'\right), \hat{\Gamma}\left(A,B,B'\right), \hat{\Theta}\left(A,B\right)\right)$$

such that:

1. Given the law of motion $\hat{\Gamma}\left(A,B,B'\right)$ and the policy rule $\hat{\Theta}\left(A,B'\right)$, $V\left(a,s,A,B,B'\right)$ is the value function solution to the program (3.4) and $a'\left(a,s,A,B,B'\right)$ is the associated saving rule,

2. Given the above value function, the maximization of the utilitarian criterion at each date is consistent with the expected policy rule $\hat{\Theta}\left(A,B\right)$:

$$\forall B, \forall A, \hat{\Theta}\left(A,B\right) = \max_{B'} \int_{a_{min}}^{a_{max}} V\left(a,s,A,B,B'\right)\Psi(a)da$$

3. Given the saving rule $a'\left(a,s,A,B,B'\right)$, and for any state of the economy (A,B), next period's mean level of asset holding defined as:

$$A' = \sum_{s\in S} \int_{a_{min}}^{a_{max}} a'\left(a,s,A,B,B'\right)\Psi\left(a\right)da$$

is consistent with the expected law of motion $\hat{\Gamma}\left(A,B,B'\right)$.

1.5 Calibration

Because of the complex dynamical behavior of the benchmark model, our calibration strategy includes an intermediate step where we first calibrate a stationary model very similar to the benchmark economy. Once the stationary model is calibrated, we use the parameters derived from this step to calibrate our benchmark economy.

We let one period in the model be one year in the data. For the sake of comparison, our main calibration target is the U.S. capital-output ratio.

We choose the corporate production function to be Cobb-Douglas:

$$Y = F(K, N) = K^{\alpha} N^{1-\alpha} \quad 0 < \alpha < 1$$

Capital share of output α is set to 0.3 and capital depreciates at a rate δ that we set to 0.075 following Aiyagari and McGrattan (1998). Household utility is of the CRRA type:

$$U(c) = \frac{c^{1-\mu}}{1-\mu}.$$

The risk aversion parameter, μ is set to 3.0. As leisure is not valued, we normalize aggregate labor supply N to unity. We also impose that households can not borrow such that $a_{min} = 0$.

The labor efficiency follows a first-order autoregressive process:

$$e_{t+1} = \rho e_t + \varepsilon_{t+1}$$

As this process is only approximated by a two states discrete process using the methodology of Adda and Cooper (2003) for simplicity, we follow the usual result in the literature of a highly persistent wage income process. Thus we set $\rho = 0.95$ and $\varepsilon = 0.2$. This yields the following transition matrix:

$$\begin{pmatrix} 0.9 & 0.1 \\ 0.1 & 0.9 \end{pmatrix}$$

In a good state, household productivity is $\overline{s} = 1.5$. In a bad state, household productivity is $\underline{s} = 0.5$. This can be interpreted as periods of employment and unemployment where a bad state corresponds to unemployment and income derives from an exogenous unemployment fund or home production.

We assume that the ratio of government purchases to GDP γ is 0.217 and the ratio of lump-sum transfers to GDP φ is 0.082 as specified in Aiyagari and McGrattan (1998). In our intermediary stationary economy, the debt over GDP ratio, noted b, is initially set to $\frac{2}{3}$, as it is the level reported by Aiyagari and McGrattan (1998) in the U.S. for the postwar period. We close the stationary model by targeting a capital-output ratio of 2.5 as reported by Aiyagari and McGrattan (1998). This target is reached by adjusting the discount rate β that we fix to a value of 0.9647.

Finally, in the benchmark model, the benevolent social planner draws with a probability λ a period when he has to choose a new level of debt. We set $\lambda = 0.1$. Table (3.1) summarizes the calibration parameters.

Table 3.1: Calibration

Parameter	Value
α	0.3
δ	0.075
μ	3
a_{min}	0
ρ	0.95
ε	0.2
γ	0.217
φ	0.082
λ	0.1
β	0.9647

2 Results

2.1 Stationary model

A stationary model is used for calibration purpose. This model only computes the long run steady state equilibrium for a given level of public debt. Households do not anticipate any change in public debt policy. In this sense, this model can be viewed as a basic derivative of the model in Aiyagari and McGrattan (1998) as the labor process is much simpler and as leisure is not valued. Our computations show that the optimal level of debt is -190% and that households would gain as much as 10.9% of consumption for being at the optimal level of public debt instead of the postwar U.S. level of 66%. In this simple model, agents do not ponder the time-consistent outcome of the economy. Instead, they believe that the debt to GDP ratio is set forever and make their choices accordingly. At the steady state equilibrium, we can compute utilitarian welfare for a given ratio of public debt and endogenously determined prices and tax rate.

To find the optimal level in such an economy, we have to iterate over several values of the debt to GDP ratio. The optimal level is the ratio that maximizes the welfare criterion. Households would prefer a level of debt rather than another because they balance the long run gains and losses of increasing public debt. A higher level of public debt increases interest rates and reduce the cost of accumulating assets to smooth consumption. As a result, a higher level of debt can be welfare increasing. On the other hand, a higher level of debt crowds out private capital and reduces consumption and decreases welfare. However, as the agents never know that the level of debt could change, we can only observe this process by exogenously setting the debt to GDP ratio and iterating on it until a ratio maximizing welfare is found.

2.2 Time-consistent equilibrium

We find that a time-consistent setting with incomplete markets and uncertainty has a strong impact on the optimal level of public debt. Our simple calibration strategy applied to the benchmark economy yields an optimal debt to GPD ratio of 210%. The level is constantly positive and large. The no-commitment solution is very different from the stationary one. In this economy, the expectations formed by agents play an important role. In a time-consistent economy, agents are fully rational. They know that the level of debt is not set forever and that it will change over time. They also predict correctly the outcomes of future choices. And finally agents realize that their individual choices can alter the evolution of the economy and future debt choices. From the point of view of agents, the economy is a sequence of choice and no choice periods drawn randomly with a known probability. At a given point in time, a choice made by an agent can alter the entire sequence. This is known and anticipated by agents. The optimal level is found when at each choice period, the level of debt desired by the economy - given by an utilitarian welfare criterion - remains constant over time.

In the economy, agents have conflictual wishes as to how the level of public debt should be, depending on the individual situation of an agent. Increasing the level of public debt has the immediate effect of reducing the tax rate. This decrease raises consumption and welfare. However, choosing a higher level of debt today means higher levels of taxes tomorrow and for the subsequent periods until a new choice can be made. Households also internalize the opposing effects of public debt found in the stationary economy. On the one hand, a higher level of public debt can help to reduce the cost of precautionary saving through higher interest rates. But on the other hand, a higher level of public debt crowds out private capital and decreases welfare. In the time-consistent equilibrium, choosing a higher level of public debt immediately raises the interest rate and crowds out private capital. However, in the subsequent periods, because the interest rate is high today, agents increase the amount of assets they hold. As a result, the interest rate is lower in the future and until a new choice can be made. We observe the opposite mechanism for the wage rate. How the effects of prices impact agents depend on the individual situation of each one. Depending on the amount of assets held by an agent, a higher interest rate today can be sought. Agents with large amounts of assets benefit from a higher interest rate today even though it will slowly decrease until a new choice can be made. The benevolent planner summarizes those gains and losses with the welfare criterion and sets the appropriate level of public debt.

3 Conclusion

This chapter uses a time-consistent framework to assess the optimal level of public debt under incomplete markets. We find the optimal level of public debt to be positive and large. Being fully rational, households internalize future policy choices and know that their own decision can affect those outcomes. Thus, the optimal level of public debt is set by a benevolent planner maximizing a welfare criterion. Each household chooses its own preferred level of public debt by comparing welfare gains today with welfare losses tomorrow and in the subsequent periods.

Conclusion

Although the level of debt has been rising since the postwar period in OECD countries, there have been little quantitative assessment of what the optimal level of debt should be and what elements impact this level. This doctoral essay introduces original frameworks to quantify the optimal level of public debt in incomplete markets economies. It also provides insight as to how this optimal level is chosen and to what extend the level of public debt influences individual decision. After a short survey of the literature in a first chapter, the second chapter reconsidered the optimal level of public debt in an economy with aggregate fluctuations. It looked at the interaction between public debt policies and the business cycle and builds a framework exhibiting aggregate fluctuations to quantify the optimal level of public debt. The main result was that the optimal level of debt is positive and higher than in a model without aggregate risk. Also, gains and losses of higher levels of public debt is different along the business cycle or across the wealth distribution. The third chapter introduced an economy where both entrepreneurial households and standard working households are subject to long run variations of the level of public debt. This chapter modeled a stylized economy with both entrepreneurial and non-entrepreneurial households subject to uninsurable risk. This setup yielded a negative steady state level of optimal public debt and matched the U.S. wealth distribution and wealth Gini. I showed that these results came from the fact that part of the accumulation behavior of working households is transferred to the entrepreneurial households. The fourth and last chapter moved away from the steady state comparison approach of the two previous chapters. We considered the importance of time consistent policies for the optimal level of debt in an incomplete market framework. We found the time-consistent optimal level of debt to be large and positive. The intuition behind this result was the following. In a time-consistent equilibrium, agents are fully rational and can predict the outcome of future debt choices. Moreover, agents make their own choices knowing the effects on the evolution of the economy. Increasing the level of debt today has the immediate effect of bringing the tax rate down. As it will boost current consumption, this is welfare increasing. But agents know that tomorrow and the consequent periods, they will have to pay higher taxes until a new choice can be made. Similarly a higher level of debt has an effect on prices. The interest rate immediately increases but falls down in the future. Higher interest rates can help agents to reduce the cost of precautionary saving and increase welfare. But as higher levels of debt crowd out private capital agents have also to endure welfare losses. Agents will choose higher levels of debt until the immediate gains are equal to future losses. Eventually, the benevolent planner will set a level of debt that maximizes total welfare in the economy.

Appendix

Appendix A

Appendix to Chapter 1

1 Computational strategy

We solve the model using the methodology developed by Den Haan (1996) and Krusell and Smith (1998). They show that agents need only a restrictive set of statistics about the wealth distribution to determine prices. This set includes the mean of the wealth distribution and the aggregate productivity shock. A linear prediction rule based only on the average level of capital provides an accurate prediction. This result comes from the near linearity of the decision rule $a'(a, s, \beta; z, \Gamma)$. As the aggregate capital stock is mainly held by rich people who have approximately the same propensity to save, next period's aggregate capital is accurately predicted by current period's aggregate capital. In our model, we assume the following law of motion:

$$\log(\bar{K}') = a_0 + a_1 z + a_2 \log(\bar{K})$$

where $\bar{K}'$ and $\bar{K}$ denote respectively the average stock of capital of the next period and of the current period. Thus the strategy is the following:

(Step 1): Given a set of parameter values (a_0, a_1, a_2) for the law of motion, we solve the individual problem. To solve the individual problem, we iterate on the Euler equation:

$$U'(c) = E\left[\beta' U'(c')(1 + r'(z', \Gamma')(1-\tau))/s, \beta; z, \Gamma\right],$$

on a discrete grid until a fix point is found. When the borrowing constraint bind, the solution can be deduced from the budget constraint.

(Step 2): Given the parameter values for individual decision rules, we solve the aggregate problem *i.e.* the coefficients of the law of motion.

(Step 3): If the parameters (a_0, a_1, a_2) found are close to the parameter values used to solve (Step 1), the algorithm has converged. Otherwise (Step 1), (Step 2) and (Step 3) are repeated until convergence.

(Step 4): Because of the aggregate fluctuations property and the government budget constraint, one more step is necessary. As the aggregate variables are not constant, even in the limit in an economy with aggregate fluctuations, we approximate a *pseudo steady state* by averaging aggregate variables over long periods of time. We first make a guess for the interest rate in this *pseudo steady state*, and with the long run budget constraint of the government, we derive a valid tax rate for this *pseudo steady state*. Also, the amount of public debt in the economy is defined according to this *pseudo steady state* by taking the appropriate ratio of the GDP in the *pseudo steady state*. Given the tax rate we just defined, we execute the above steps 1, 2 and 3. When those steps have converged, we update our guess of the interest rate in the *pseudo steady state* until a fix point is found.

When step 4 is complete, the *pseudo steady state* interest rate coincides with the average interest rate in the actual economy with aggregate fluctuations, the government budget constraint is balanced and the lump sum transfer to individuals amounts to zero at the equilibrium. All results in the model are derived from simulated data. The simulated sample consist of 11000 periods and the first 1000 are discarded. The distribution is approximated by a sample of 30000 households at each period.

2 Calibration

We now show how we derived the transition matrices for aggregate state changes (η), for joint transition between aggregate states and labor market statuses (Π) and for discount factor changes (Υ).

2.1 Aggregate state change transition matrix

To deduce the aggregate state change transition matrix η, we solve the following system:

$$\left.\begin{aligned} \eta_{gg} &= \eta_{bb} \\ \eta_{bg} &= \eta_{gb} \\ \eta_{gg} + \eta_{gb} &= 1 \\ \eta_{bb} + \eta_{bg} &= 1 \\ \eta_{bg} &= \frac{1}{8} \end{aligned}\right\} \Longrightarrow \begin{pmatrix} 0.875 & 0.125 \\ 0.125 & 0.875 \end{pmatrix}$$

As we assumed that the duration of a boom or a recession is the same, we deduce the two first equations. Moreover, the duration of a cycle is set to 8 quarters, it follows that $\eta_{bg} = Pr(z_{t+1} = g/z_t = b) = \frac{1}{8}$ and $\eta_{gb} = Pr(z_{t+1} = b/z_t = g) = \frac{1}{8}$.

2.2 Matrix for joint transition between aggregate states and labor market statuses

The determination of the matrix Π that describes the transition between unemployment and employment requires the identification of the aggregate shock (whether we are in a recession or in a boom). The transition matrix Π is built thanks to the matrix η and to the transition matrixes Π^{gg}, Π^{bb} ,Π^{gb} and Π^{bg}. Π verifies:

$$\Pi = \begin{pmatrix} \eta_{bb}\Pi^{bb} & \eta_{bg}\Pi^{bg} \\ \eta_{gb}\Pi^{gb} & \eta_{gg}\Pi^{gg} \end{pmatrix}$$

We assumed that in recessions the duration of the unemployment, that we note $durub$, amounts to 2.5 quarters and the unemployment rate u_b is set to 10%. In booms, the duration of unemployment, $durug$, is equal to 1.5 quarters and the unemployment rate, u_g, is set to 4%. From this information, we can deduce the matrices Π^{gg} and Π^{bb}.

The transition matrix Π^{gg} corresponds to the case $(z, z') = (g, g)$. It verifies:

$$\Pi^{gg} = \begin{pmatrix} \Pi^{gg}_{uu} & \Pi^{gg}_{ue} \\ \Pi^{gg}_{eu} & \Pi^{gg}_{ee} \end{pmatrix}$$

Solving the system below gives the values of Π^{gg}_{ee}, Π^{gg}_{eu}, Π^{gg}_{ue} and Π^{gg}_{uu}:

$$\begin{cases} \Pi^{gg}_{ee} + \Pi^{gg}_{eu} = 1 \\ \Pi^{gg}_{ue} + \Pi^{gg}_{uu} = 1 \\ \Pi^{gg}_{ue} = \dfrac{1}{durug} \\ \Pi^{gg}_{ee} = 1 - \dfrac{u_g \Psi^{gg}_{ue}}{1 - u_g} \end{cases} \Longrightarrow \Pi^{gg} = \begin{pmatrix} 0.3333 & 0.6667 \\ 0.0278 & 0.9722 \end{pmatrix}$$

The transition matrix Π^{bb} corresponds to the case $(z, z') = (b, b)$. It verifies:

$$\Pi^{bb} = \begin{pmatrix} \Pi^{bb}_{uu} & \Pi^{bb}_{ue} \\ \Pi^{bb}_{eu} & \Pi^{bb}_{ee} \end{pmatrix}$$

Solving the system below gives the values of Π^{bb}_{ee}, Π^{bb}_{eu}, Π^{bb}_{ue} and Π^{bb}_{uu}:

$$\begin{cases} \Pi^{bb}_{ee} + \Pi^{bb}_{eu} = 1 \\ \Pi^{bb}_{ue} + \Pi^{bb}_{uu} = 1 \\ \Pi^{bb}_{ue} = \dfrac{1}{durub} \\ \Pi^{bb}_{ee} = 1 - \dfrac{u_b \Pi^{bb}_{ue}}{1-u_b} \end{cases} \Longrightarrow \Pi^{bb} = \begin{pmatrix} 0.6 & 0.4 \\ 0.0445 & 0.9555 \end{pmatrix}$$

When the cycle changes the unemployment rate changes. The transitions between unemployment and employment get modified. We make the same assumptions as Krusell and Smith (1998):

$$\begin{cases} \Pi^{bg}_{uu} = \Pr(\epsilon_{t+1} = u^g / \epsilon_t = u^b) = 0.75 \Pi^{gg}_{uu} \\ \Pi^{gb}_{uu} = \Pr(\epsilon_{t+1} = u^b / \epsilon_t = u^g) = 1.25 \Pi^{bb}_{uu} \end{cases}$$

The probability to remain unemployed when the next period is a recession (resp. boom), increases (resp. decreases) since by assumption the unemployment rate is higher in recession than in boom.

The transition matrix Π^{bg} corresponds to the case $(z, z') = (b, g)$. It verifies:

$$\Pi^{bg} = \begin{pmatrix} \Pi^{bg}_{uu} & \Pi^{bg}_{ue} \\ \Pi^{bg}_{eu} & \Pi^{bg}_{ee} \end{pmatrix}$$

The system below gives us Π^{bg}_{ee}, Π^{bg}_{eu}, Π^{bg}_{ue} and Π^{bg}_{uu}:

$$\begin{cases} \Pi^{bg}_{ee} + \Pi^{bg}_{eu} = 1 \\ \Pi^{bg}_{ue} + \Pi^{bg}_{uu} = 1 \\ \Pi^{bg}_{uu} = 0.75\Pi^{gg}_{uu} \\ \Pi^{bg}_{ee} = \dfrac{((1-u_g) - u_b\Pi^{bg}_{ue})}{1-u_b} \end{cases} \Longrightarrow \Pi^{bg} = \begin{pmatrix} 0.25 & 0.75 \\ 0.0167 & 0.9833 \end{pmatrix}$$

The transition matrix Π^{gb} corresponds to the case $(z, z') = (g, b)$.

$$\mathbf{\Pi}^{gb} = \begin{pmatrix} \Pi^{gb}_{ee} & \Pi^{gb}_{eu} \\ \Pi^{gb}_{ue} & \Pi^{gb}_{uu} \end{pmatrix}$$

The system below gives us Π^{gb}_{ee}, Π^{gb}_{eu}, Π^{gb}_{ue} and Π^{gb}_{uu}:

$$\begin{cases} \Pi^{gb}_{ee} + \Pi^{gb}_{eu} = 1 \\ \Pi^{gb}_{ue} + \Pi^{gb}_{uu} = 1 \\ \Pi^{gb}_{uu} = 1.25\Pi^{bb}_{uu} \\ \Pi^{gb}_{ee} = \dfrac{((1-u_b) - u_g\Pi^{gb}_{ue})}{1-u_g} \end{cases} \Longrightarrow \Pi^{gb} = \begin{pmatrix} 0.75 & 0.25 \\ 0.0729 & 0.9271 \end{pmatrix}$$

2.3 Matrix for discount factor changes

We assumed that the discount factors follow a three-states first-order Markov process. Therefore, the matrix describing the transition from the discount factor β_i to the discount factor β_j is the following:

$$\Upsilon = \begin{pmatrix} \Upsilon_{ll} & \Upsilon_{lm} & \Upsilon_{lh} \\ \Upsilon_{ml} & \Upsilon_{mm} & \Upsilon_{mh} \\ \Upsilon_{hl} & \Upsilon_{hm} & \Upsilon_{hh} \end{pmatrix}$$

As we assumed that there is no immediate transition between β_l and β_h as in Krusell and Smith (1998), it involves that $\Upsilon_{lh} = \Upsilon_{hl} = 0$. Moreover, as we set the duration of the extreme states (β_l and β_h) to 50 years namely 200 quarters, we have $\Upsilon_{lm} = \frac{1}{200} = \Upsilon_{hm}$. Solving the following system gives us the transition matrix Υ:

$$\left.\begin{array}{c} \Upsilon_{ll} + \Upsilon_{lm} + \Upsilon_{lh} = 1 \\ \Upsilon_{ml} + \Upsilon_{mm} + \Upsilon_{mh} = 1 \\ \Upsilon_{hl} + \Upsilon_{hm} + \Upsilon_{hh} = 1 \\ \Upsilon_{lh} = \Upsilon_{hl} = 0 \\ \Upsilon_{lm} = \frac{1}{200} = \Upsilon_{hm} \\ \Upsilon_{ml} = \frac{\Pr(\beta_t=\beta_l)\Upsilon_{lm}}{\Pr(\beta_t=\beta_m)} \\ \Upsilon_{mh} = \frac{\Pr(\beta_t=\beta_h)\Upsilon_{hm}}{\Pr(\beta_t=\beta_m)} \end{array}\right\} \Longrightarrow \Upsilon = \begin{pmatrix} 0.995 & 0.005 & 0 \\ 0.0007 & 0.9979 & 0.0014 \\ 0 & 0.005 & 0.995 \end{pmatrix}$$

3 Model without aggregate risk

We now briefly detail the model without aggregate risk and its calibration. Most of this model is similar to the benchmark model, thus we underline only differences.

Model

As this model serves comparison purposes most of the benchmark assumptions remain unchanged. The assumptions about the representative firm are similar with the exception of the technical progress z. In the absence of aggregate risk, z is fixed to its average value. In the absence of aggregate risk, the budget constraint of the government is:

$$G_t + r_t B_t = B_{t+1} - B_t + T_t$$

Household preferences are unchanged. In the absence of aggregate risk, the matrix that describes the transition on the labor market becomes:

$$\pi = \begin{pmatrix} \pi_{uu} & \pi_{ue} \\ \pi_{eu} & \pi_{ee} \end{pmatrix}$$

with $\pi_{uu} = \Pr(s_{t+1} = u | s_t = u)$. When there is no aggregate risk, it is no longer necessary to distinguish the nature of the cycle. In the absence of aggregate risk, the state variables are summarized by the vector (a, s, β). The program the household solves is:

$$v(a, s, \beta) = \max_{c,\, a'} \{u(c) + \beta E\left[v(a', s', \beta') | (s, \beta)\right]\} \tag{A.1}$$

subject to:

$$c + a' = (1 + r(1-\tau))a + w\chi(s) \tag{A.2}$$

$$c \geq 0 \tag{A.3}$$

$$a' \geq 0 \tag{A.4}$$

with

$$\chi\left(s\right) = \begin{cases} \theta \text{ if } s = u, \\ (1-\tau) \text{ if } s = e \end{cases}$$

Equilibrium

The recursive equilibrium consists of a set of decision rules for consumption and asset holding $\{c(a, s,), a'(a, s, \beta; z, \Gamma)\}$, aggregate capital K, factor prices $\{r, w\}$, tax rate τ satisfying these conditions:

1. Given the prices $\{r, w\}$, the decision rules $\{c(a, s, \beta), a'(a, \epsilon, \beta)\}$ solve the dynamic programming problem (A.1) subject to the constraints (A.2), (A.3) and (A.4)

2. Market price arrangements are:

$$\begin{aligned} r &= \alpha z K^{\alpha-1} N^{1-\alpha} - \delta \\ w &= (1-\alpha) z K^{\alpha} N^{-\alpha} \end{aligned}$$

3. Government budget constraint is balanced.

4. Capital Market clears when:

$$K + B = \int a'(a, \epsilon, \beta) d\Gamma(a, s, \beta)$$

   with $\Gamma(a, s, \beta)$ the distribution of agents over asset holdings, employment status and preferences.

Calibration

We present here the calibration strategy in an economy without aggregate risk. The calibration of the preferences, the discount factors and the behavior of the government are unmodified. The calibration of z and the characteristics of the labor market differ in the absence of aggregate risk. In an economy without aggregate risk, z is constant and set to the average value of the aggregate shock, namely the unit value. As we consider three alternative calibrations of the idiosyncratic labor process, we detail them sequentially.

In the Imrohoroglu method, the unemployment rate u and the unemployment spells $duru$ are respectively set to 7% and 2 quarters (we average over good and bad periods in the benchmark model). These two assumptions define the transition matrix π:

$$\begin{cases} \pi_{uu} + \pi_{ue} = 1 \\ \pi_{eu} + \pi_{ee} = 1 \\ \pi_{ue} = \dfrac{1}{duru} \\ \pi_{ee} = 1 - \dfrac{u\pi_{ue}}{1-u} \end{cases} \Longrightarrow \begin{pmatrix} 0.5 & 0.5 \\ 0.0376 & 0.9624 \end{pmatrix}$$

In the brute-force averaging method, we first simulate the transition process and find the transition probabilities. This yields the following transition matrix:

$$\begin{pmatrix} \pi_{uu} & \pi_{ue} \\ \pi_{eu} & \pi_{ee} \end{pmatrix} = \begin{pmatrix} 0.50749 & 0.49251 \\ 0.03708 & 0.96292 \end{pmatrix}$$

With this matrix, we derive an unemployment rate of 7%.

In the conditional approach, we use our definitions of transition probabilities to derive the following transition matrix:

$$\begin{pmatrix} \pi_{uu} & \pi_{ue} \\ \pi_{eu} & \pi_{ee} \end{pmatrix} = \begin{pmatrix} 0.53423 & 0.46577 \\ 0.03685 & 0.96315 \end{pmatrix}$$

With this matrix, we derive an unemployment rate of 7.33%.

Appendix B

Appendix to Chapter 2

1 Computational strategy

The model is solved using the following steps:

1. We guess values for the equilibrium interest rate, the equilibrium amount of hours worked along with values for the aggregate level of risky assets and production of entrepreneurs. Those guesses yield the steady state tax rate.

2. Given prices and the tax rate, we solve the problem of the entrepreneur and the non-entrepreneur by iterating on each agent's consumption policy function.

3. We iterate on the distribution of agents until a stationary distribution is found.

4. Aggregate variables obtained with the stationary distribution are used to update our guesses. The process is repeated until a fixed point for each guessed

element is found.

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[1]This chapter is a joint work with Audrey Desbonnet

[2]This chapter is a joint work with Audrey Desbonnet and Thomas Weitzenblum